SECTION I (50 marks)

Answer all the questions in this section in the spaces provided. **14/11/2024**

1. An arithmetic progression (AP) is given as $600+650+700+750+...$ Determine:

(a) the 30th term of the AP; (2 marks)

$T_n = a+(n-1)d$

$a = 600,\ n = 30$

$d = 700-650 = 50$ ✓

$T_{30} = 600+50(30-1)$

$= 60+(29\times 50)$

$= 2050$ ✓

(b) the sum of the first 30 terms of the AP. (2 marks)

$S_n = \frac{n}{2}[a+l];\ l = \textit{last term}\ \{30^{th}\ \textit{term}\}$

$S_{30} = \frac{30}{2}[600+2050]$ ✓

$= 15\times 2650$

$= 39750$ ✓

Alternatively,

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{30} = \frac{30}{2}[(600\times 2)+50(30-1)]$ ✓

$= 15(1200+1450)$

$= 39750$ ✓

2. The quadratic equation $5x^2+kx+20=0$ has only one root. Determine the possible values of *k*. (2 marks)

For a repeated root the discriminant, $b^2-4ac=0$ hence $b^2=4ac$.

$\therefore k^2 = 4\times 5\times 20$ ✓

$k^2 = 400$

$k = \pm\sqrt{400} = \pm 20\quad (20\ \ or\ \ -20)$ ✓

Confidential

3. Without using mathematical tables or a calculator, evaluate $\dfrac{\log 125+\log 64}{\log\sqrt[6]{5}+\log\sqrt[3]{2}}$. (3 marks)

$125 = 5^3,\ 64 = 2^6,\ \sqrt[6]{5} = 5^{\frac{1}{6}},\ \sqrt[3]{2} = 2^{\frac{1}{3}}$

$$\frac{\log(5^3\times 2^6)}{\log\left(5^{\frac{1}{6}}\times 2^{\frac{1}{3}}\right)} ✓ = \frac{\log(5^1\times 2^2)^3}{\log(5^1\times 2^2)^{\frac{1}{6}}} = \frac{3\log\left(\cancel{5^1\times 2^2}\right)^1}{\frac{1}{6}\log\left(\cancel{5^1\times 2^2}\right)^1} ✓ = \frac{3}{\frac{1}{6}} = 3\times\frac{6}{1} = 18 ✓$$

Alternatively

$$\frac{\log(5^3\times 2^6)}{\log\left(5^{\frac{1}{6}}\times 2^{\frac{1}{3}}\right)} ✓ = \frac{\log 5^3+\log 2^6}{\log 5^{\frac{1}{6}}+\log 2^{\frac{1}{3}}} = \frac{3\log 5+6\log 2}{\frac{1}{6}\log 5+\frac{1}{3}\log 2} ✓ = \frac{3\left(\cancel{\log 5+2\log 2}\right)^1}{\frac{1}{6}\left(\cancel{\log 5+2\log 2}\right)^1} = \frac{3}{\frac{1}{6}} = 3\times\frac{6}{1} = 18 ✓$$

4. Make *x* the subject of the formula $y = \dfrac{a}{b^x}$. (3 marks)

$yb^x = a \Rightarrow \boxed{b^x = \frac{a}{y}}$ ✓

Taking logs on both sides:

$x\log b = \log\left(\frac{a}{b}\right)$ ✓ hence $\boxed{x = \dfrac{\log\left(\frac{a}{b}\right)}{\log b}}$ ✓

Alternatively

$yb^x = a \Rightarrow \boxed{b^x = \frac{a}{y}}$ ✓

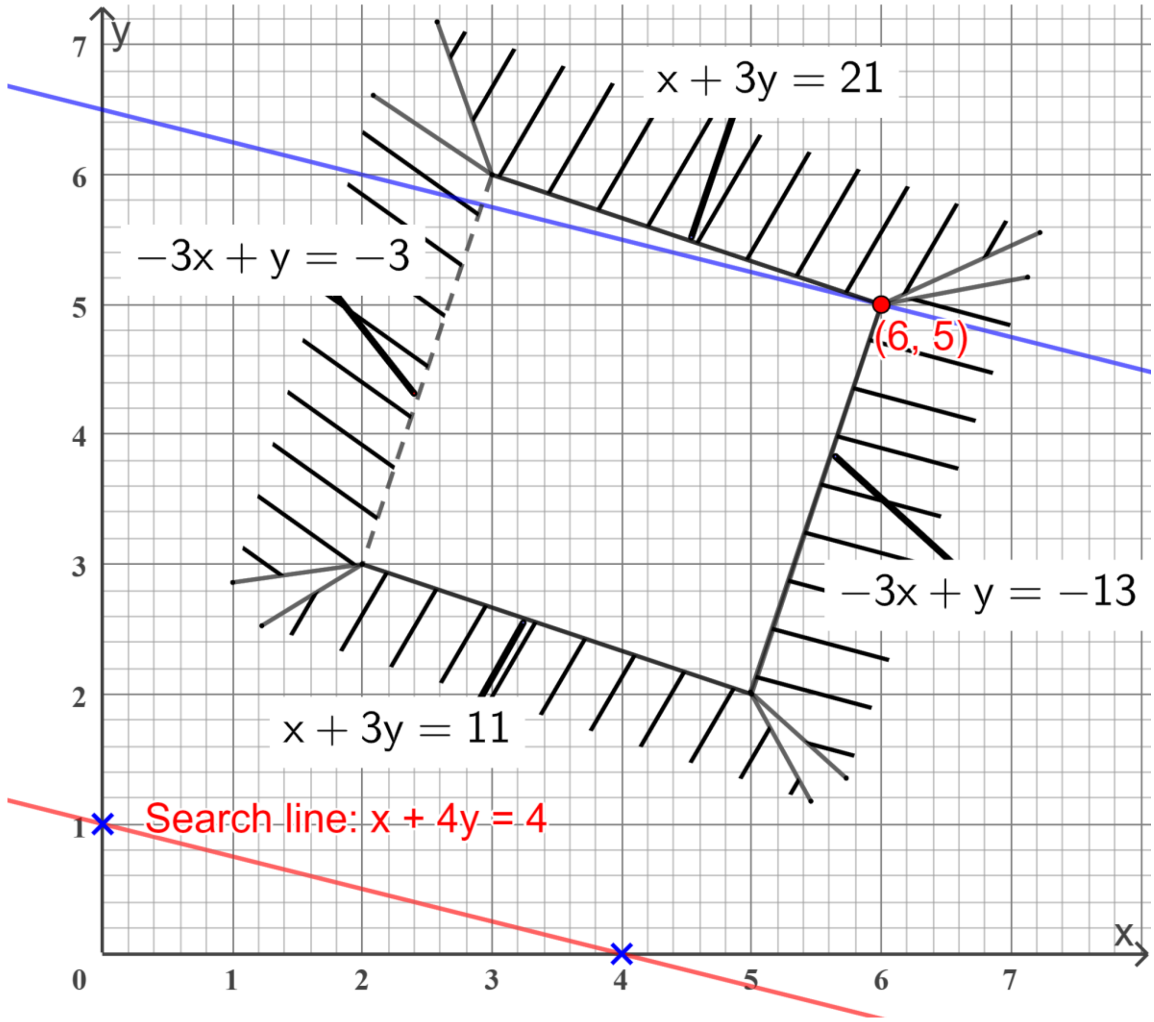
Taking logs on both sides:

$x\log b = \log\left(\frac{a}{b}\right)$

$x\log b = \log a - \log b$ ✓

$\boxed{x = \dfrac{\log a - \log b}{\log b}}$ ✓ or $\boxed{x = \dfrac{\log a}{\log b} - 1}$ ✓

5. The unshaded region on the Cartesian plane satisfies the inequalities $x+3y \le 21, -3x+y < -3, -3x+y \ge -13$ and $x+3y \ge 11$.



Find the maximum value of $(x+4y)$ for the integral coordinates $P(x, y)$ lying in the unshaded region.

(3 marks)

Objective function: $\boxed{x+4y=\mathbf{M}}$ **Let** $x+4y=4$. **Mark and join points (4, 0) and (0, 1) to obtain the search line.** ✓ **Using a ruler and set square only, produce a parallel line to the search line at the last plausible point (6, 5)** ✓ **before leaving the feasible region.** $\mathbf{M}=6+(4\times5)=26$ ✓

6. An aircraft flew due west from point $A(39.64°N, 50°E)$ to $B(39.64°N, 20°W)$. Calculate the distance covered by the aircraft correct to the nearest km. $\left(\text{Take } \pi=\frac{22}{7} \text{ and } R=6370 \text{ km}\right)$ (3 marks)

Longitude difference, $\theta = 50°+20°=70°$ ✓ (A and B are on either side of the prime meridian)

$\beta = \textit{latitude angle}, R = \textit{Radius of the earth} = 6370\,km$

$$\textit{Distance along a parallel of latitude} = \frac{\theta}{360°}\times 2\pi R\cos\beta$$

$$= \frac{70°}{360°}\times 2\times\frac{22}{7}\times 6370\times\cos 39.64°$$ ✓

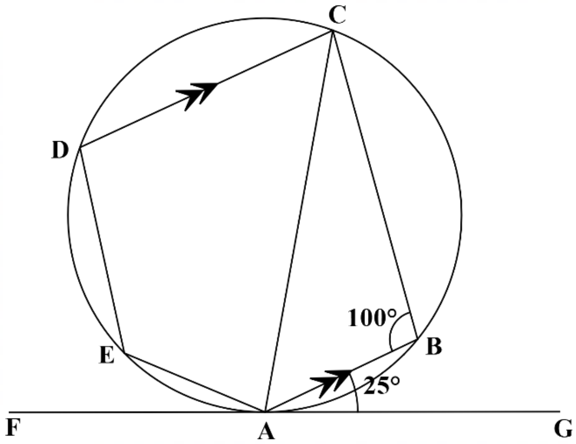
$$= \frac{70070}{9}\times 0.7701$$

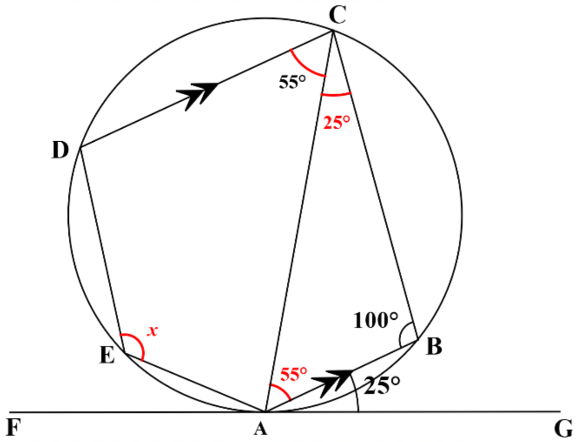
$$= 5995.6563\,km$$

$\approx$ **5996 km** ✓

✔ Completed

7. In the following figure; A, B, C, D and E are points on the circumference of the circle. Line AB is parallel to line DC and line FAG is tangent to the circle at A. Angle GAB = 25° and $\angle ABC = 100°$.





Determine the size of:

(a) $\angle$BAC; (1 mark)

$\angle$**BCA** = $\angle$**GAB** = 25° (by alternate segment//tangent-chord theorem)

$180° - (100° + 25°) = \mathbf{55°}$✓

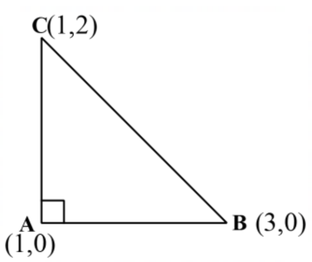
(b) $\angle$AED. (2 marks)

Since AB || CD, AC is a transversal. Thus, $\angle BAC = \angle ACD = 55°$ (*alternate angles*)✓

Opposite angles of cyclic quadrilateral ACDE are supplementary (add up to 180°.)

$\therefore x = 180° - 55° = \mathbf{125°}$✓

8. The triangle ABC with vertices $A(1,0), B(3,0)$ and $C(1,2)$ is transformed by the matrix $\mathbf{T} = \begin{pmatrix} 3k & 1.6 \\ 3k & -0.9 \end{pmatrix}$ onto triangle $A'B'C'$. Given that the area of triangle $A'B'C'$ is 6 square units, determine the value of k. (3 marks)



$$\text{Area of } \triangle ABC = \frac{1}{2} \times 2 \times 2$$
$$= 2 \ sq.units$$

$$ASF = \frac{\text{Area of } \triangle A'B'C'}{\text{Area of } \triangle ABC}$$
$$= \frac{6}{2}$$
$$= 3$$ ✓

$$\text{Det } \mathbf{T} = \begin{vmatrix} 3k & 1.6 \\ 3k & -0.9 \end{vmatrix} = 3k(-0.9) - 3k(1.6)$$
$$= -2.7k - 4.8k$$
$$= -7.5k$$ ✓

Det **T** = ASF hence $-7.5k = 3$

$k = \frac{3}{-7.5} = -\frac{\mathbf{1}}{\mathbf{4}}$ or $\mathbf{-0.25}$✓

9. Solve the equation $6\cos^2 x + \sin x = 4$ for $0° \le x \le 180°$, giving the answer correct to 2 decimal places. (3 marks)

$sin^2 x + cos^2 x = 1 \Rightarrow \boxed{cos^2 x = 1 - sin^2 x}$

$6(1 - \sin x^2) + \sin x = 4$

$6 - 6\sin x^2 + \sin x = 4$

$6\sin x^2 - \sin x - 2 = 0$✓

$6\sin x^2 + 3\sin x - 4\sin x - 2 = 0$

$3\sin x(2\sin x + 1) - 2(2\sin x + 1) = 0$

$(3\sin x - 2)(2\sin x + 1) = 0$✓

$3\sin x - 2 = 0$ or $2\sin x + 1 = 0$

$3\sin x = 2$ $\quad$ $2\sin x = -1$

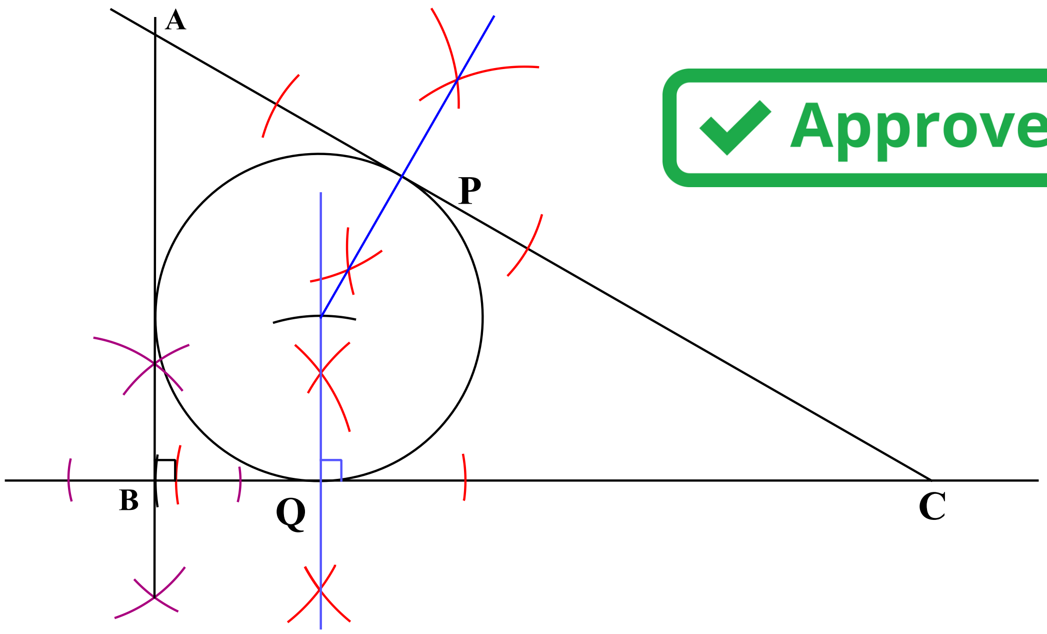
$\sin x = 0.6667$ $\quad$ $\sin x = -0.5$

Since sine is positive in the range $0° \le x \le 180°$;

$x = \sin^{-1} 0.6667$

$= \mathbf{41.81°, 138.19°}$✓

10. The figure below shows a circle. Lines CP and CQ are tangents to the circle at points P and Q respectively.



The circle is to be inscribed in a triangle ABC. Point B lies on CQ produced and $\angle CBA = 90°$. Use a ruler and a pair of compasses only to:

(a) locate point O, the centre of the circle; (2 marks)

(b) complete triangle ABC. (2 marks)

11. The deviations of the masses of 10 students from an assumed mean are:

$$-10, -5, -2, 1, 4, 5, 7, 8, 9, 13$$

The mass of the heaviest student was 58 kg. Calculate the mean mass of the students. (3 marks)

Let the assumed mean be A. $\therefore\ 58 - A = 13 \Rightarrow A = 58 - 13 = 45$✓

t	-10	-5	-2	1	4	5	7	8	9	13	
f	1	1	1	1	1	1	1	1	1	1	$\sum f = 10$
ft	-10	-5	-2	1	4	5	7	8	9	13	$\sum ft = 30$✓

$$\bar{x} = A + \frac{\sum ft}{\sum f} = 45 + \frac{30}{10} = \mathbf{48}$$✓

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12. The following table shows part of a monthly income tax rates for a certain year.

Monthly taxable income (Ksh.)	**Tax rate (%)**
0 to 11 180	10
11 181 to 21 714	15
21715 to 32 248	20

In a certain month an employee paid a net tax of Ksh. 2 200 after getting a tax relief of Ksh. 1 280. Calculate the employee's taxable income that month. (3 marks)

Gross tax : $\left(\frac{10}{100}\times 11180\right) + \left(\frac{15}{100}\times 10534\right) + \left(\frac{20}{100}\times y\right) = 2200 + 1280$✓

$$1118 \quad + \quad 1580.10 \quad + \quad 0.2y \quad = 3480$$

$$0.2y = 781.9$$

$$y = \text{Ksh. } 3909.50$$✓

Taxable income $= \text{Ksh. } (21714 + 3909.50) =$ **Ksh. 25 623.50**✓

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13. The equation of a circle is given by $x^2+y^2-3x+4y=0$. Determine:

(a) the coordinates of the centre of the circle; (2 marks)

Compare $x^2+y^2-3x+4y=0$ *with* ↓

$x^2+y^2-2ax-2by+\boxed{a^2+b^2-r^2}=0$

By comparing: $-2a=-3 \Rightarrow a=\frac{-3}{-2}=1.5$

$-2b=4 \quad \Rightarrow b=-\frac{4}{2}=-2$ ✓

Centre $(1.5,-2)$ ✓

Alternatively

Rearranging: $x^2-3x+y^2+4y=0$

$x^2-3x+\left(\frac{-3}{2}\right)^2+y^2+4y+\left(\frac{4}{2}\right)^2=\left(\frac{-3}{2}\right)^2+\left(\frac{4}{2}\right)^2$ ✓

$(x-1.5)^2+(y+2)^2=6.25$

$(x-a)^2+(y-b)^2=r^2$

$-a=-1.5 \Rightarrow a=1.5, \quad -b=2 \Rightarrow b=-2$

Centre $(1.5,-2)$ ✓

(b) the area of the circle in terms of π. (1 mark)

$r^2=6.25$ hence *area of circle* $=\pi r^2=$ **6.25π sq.units** ✓

or using $a^2+b^2-r^2=c,\ r^2=a^2+b^2-c$

$r^2=1.5^2+(-2)^2-0$

$=6.25$

Area of circle $=\pi r^2=$ **6.25π sq.units** ✓

14. The position vectors of points A, B and C are such that $\mathbf{OA}=3\mathbf{i}-5\mathbf{j}-4\mathbf{k}, \mathbf{OB}=\mathbf{j}+8\mathbf{k}$ and $\mathbf{OC}=-2\mathbf{i}+5\mathbf{j}+16\mathbf{k}$. Show that the points A, B and C are collinear. (3 marks)

Let **AC** and **AB** be 2 vectors such that $\mathbf{AC}=k\mathbf{AB}$.

$\mathbf{AC}=\mathbf{OC}-\mathbf{OA}=(-2\mathbf{i}+5\mathbf{j}+16\mathbf{k})-(3\mathbf{i}-5\mathbf{j}-4\mathbf{k})$

$=-5\mathbf{i}+10\mathbf{j}+20\mathbf{k}$

$\mathbf{AB}=\mathbf{OB}-\mathbf{OA}=(\mathbf{j}+8\mathbf{k})-(3\mathbf{i}-5\mathbf{j}-4\mathbf{k})$ ✓

$=-3\mathbf{i}+6\mathbf{j}+12\mathbf{k}$

$-5\mathbf{i}+10\mathbf{j}+20\mathbf{k}=k(-3\mathbf{i}+6\mathbf{j}+12\mathbf{k})$

$k=\frac{-5}{-3}=\frac{10}{6}=\frac{20}{12}=\frac{5}{3}$ ✓

$\mathbf{AC}=\frac{5}{3}\mathbf{AB}$ hence $\mathbf{AC} \parallel \mathbf{A}$ and A is a common point therefore A, B and C are collinear. ✓

Alternatively

$\mathbf{AC}=\begin{pmatrix}-2\\5\\16\end{pmatrix}-\begin{pmatrix}3\\-5\\-4\end{pmatrix}=\begin{pmatrix}-5\\10\\20\end{pmatrix}$

$\mathbf{AB}=\begin{pmatrix}0\\1\\8\end{pmatrix}-\begin{pmatrix}3\\-5\\-4\end{pmatrix}=\begin{pmatrix}-3\\6\\12\end{pmatrix}$ ✓

$\begin{pmatrix}-5\\10\\20\end{pmatrix}=k\begin{pmatrix}-3\\6\\12\end{pmatrix} \Rightarrow k=\frac{-5}{-3}=\frac{10}{6}=\frac{20}{12}=\frac{5}{3}$ ✓

$\mathbf{AC}=\frac{5}{3}\mathbf{AB}$ hence $\mathbf{AC} \parallel \mathbf{A}$ and A is a common point therefore A, B and C are collinear. ✓

15. A particle starts from point O and moves in a straight line so that its velocity v ms^{-1} after time t seconds is given by $v=9t^2-18t+10$. Calculate the distance covered by the particle between the time $t=1$ second and $t=2$ seconds. (3 marks)

$S=\int_1^2 (9t^2-18t+10)\,dt$

$=\left[\frac{9t^{2+1}}{3}-\frac{18t^{1+1}}{2}+\frac{10t^{0+1}}{0+1}\right]_1^2$ ✓

$=\left[3t^3-9t^2+10t\right]_1^2$

$=\left[3(2^3)-9(2^2)+10(2)\right]-\left[3(1^3)-9(1^2)+10(1)\right]$ ✓

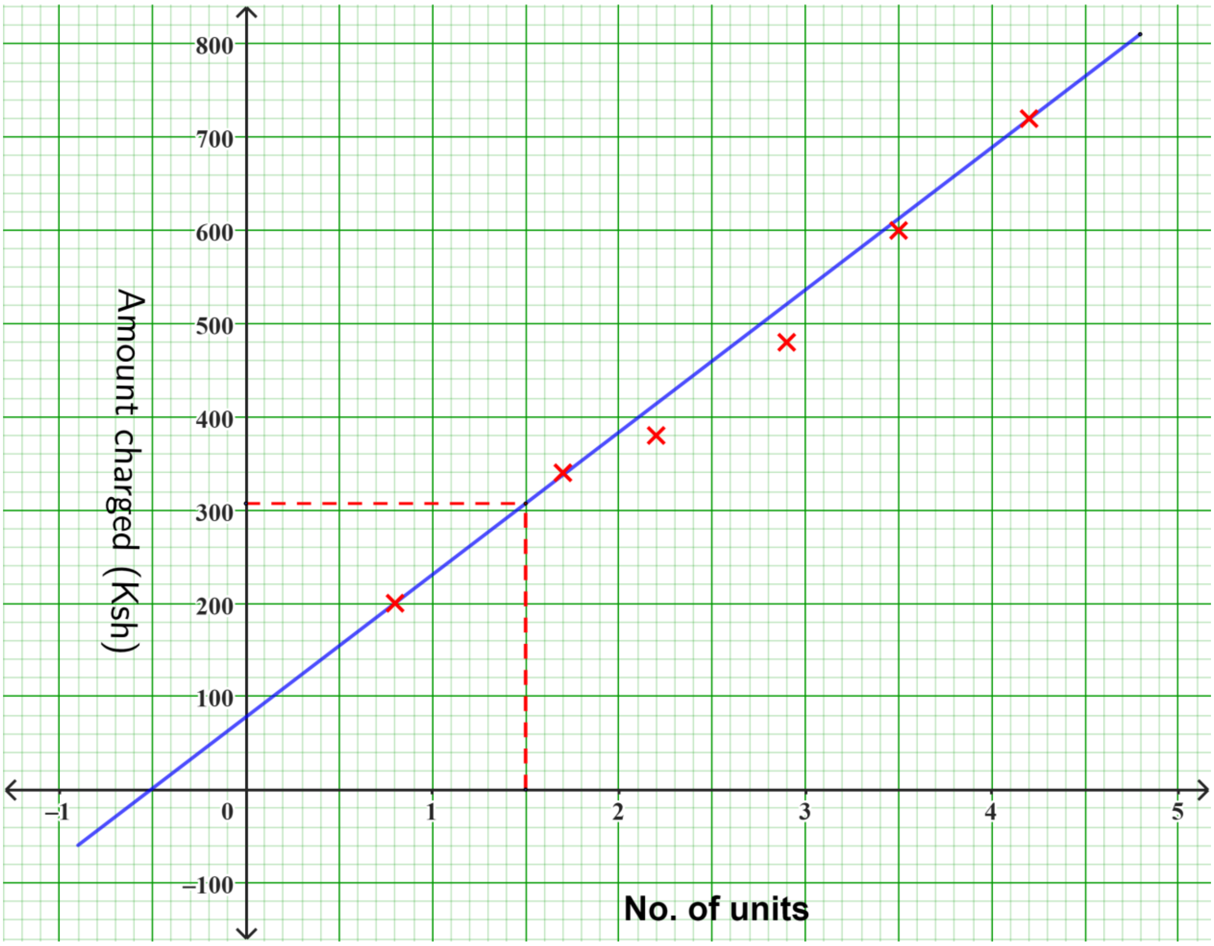
$=8-4$

$=$ **4 metres** ✓

16. The following table shows the number of units (U) of water consumed by 6 households in a month. The corresponding amount (A) charged is also given.

No. of units (U)	0.8	1.7	2.2	2.9	3.5	4.2
Amount (A) charged in Ksh.	200	340	380	480	600	720

(a) Using the scale 2 cm to represent 1 unit on the x-axis and 1 cm to represent Ksh. 100 on the y-axis, draw the line of best fit for the data on the grid provided. (3 marks)



(b) Estimate the cost of 1.5 units of water. (1 mark)

≈ **Ksh. 310**✓

SECTION II (50 marks)

*Answer only **five** questions in this section in the spaces provided.*

17. A poultry dealer has to types of chicken feeds: type A and type B. He sells 1 kg of type A at Ksh. 45 and 1 kg type B at Ksh. 30. He makes a profit of 20% per kg of type A feed sold and 25% per kg of typer B feed sold. He also sales mixtures of type A and type B feeds.

(a) Determine the amount of profit made by the dealer for selling 1 kg of:

(i) type A feed. (1 mark)

$$\text{Profit} = \frac{20}{120} \times 45 = \underline{\underline{\textbf{Ksh. 7.50}}} \checkmark \qquad \text{OR} \qquad \text{Profit} = 45 - \left(\frac{100 \times 45}{120}\right) = \underline{\underline{\textbf{Ksh. 7.50}}} \checkmark$$

(ii) type B feed. (1 mark)

$$\text{Profit} = \frac{25}{125} \times 30 = \underline{\underline{\textbf{Ksh. 6}}} \checkmark \qquad \text{OR} \qquad \text{Profit} = 30 - \left(\frac{100 \times 30}{125}\right) = \underline{\underline{\textbf{Ksh. 6}}} \checkmark$$

(b) Type A and type B feeds were mixed in the ratio $3:7$. Calculate:

(i) the selling price of 1 kg of the mixture; (2 marks)

$$\underset{\text{(per kg of mixture)}}{\text{Selling price}} = \frac{(3 \times 45) + (7 \times 30)}{3 + 7} \checkmark = \frac{345}{10}$$

$$= \underline{\underline{\textbf{Ksh. 34.50}}} \checkmark$$

(ii) the profit made by the dealer in selling 50 kg of the mixture. (2 marks)

$$\text{Profit in selling 1 kg of mixture} = \frac{(3 \times 7.50) + (7 \times 6)}{10} = \text{Ksh. 6.45} \checkmark$$

$$\text{Proft in selling 50 kg of mixture} = 50 \times 6.45 = \underline{\underline{\textbf{Ksh. 322.50}}} \checkmark$$

(c) The dealer made a profit of Ksh. 1 387.50 for the sale of 200 kg of a different mixture of type A and type B feeds. Determine the ratio of type A feed to that of type B feed in the mixture. (4 marks)

Let the ratio of A:B in the mixture be $x : y$.

$$200\left(\frac{7.50x + 6y}{x + y}\right) = 1387.50 \checkmark$$

Dividing both sides by 200...

$$\frac{7.50x + 6y}{x + y} = \frac{111}{16}$$

$$16(7.50x + 6y) = 111(x + y) \checkmark$$

$$120x + 96y = 111x + 111y$$

$$9x = 15y \checkmark$$

Dividing both sides by $9y$...

$$\frac{x}{y} = \frac{5}{3}$$

$$\therefore \underline{\underline{\mathbf{A : B = 5 : 3}}} \checkmark$$

18.

(a) A quantity P is partly constant and partly varies as the square root of a quantity Q. Given that P = 20 when Q = 4 and that P = 60 when Q = 100, find Q when P = 22. (4 marks)

$P = C + k\sqrt{Q}$ $20 = C + k\sqrt{4} \Rightarrow 20 = C + 2k \quad ...(i)$ $60 = C + k\sqrt{100} \Rightarrow 60 = C + 10k \quad ...(ii)$ ✓ $(ii) - (i): \; 8k = 40$ $k = 5$ *Using* (i), $C = 20 - (2 \times 5) = 10$ $\therefore \boxed{P = 10 + 5\sqrt{Q}}$ ✓	When $P = 22$; $\quad 5\sqrt{Q} = 22 - 10$ ✓ $5\sqrt{Q} = 12$ $\sqrt{Q} = 2.4$ $Q = 2.4^2$ $= \mathbf{5.76}$ ✓

(b) Three quantities, T, U and V are such that T varies directly as the square of $(10 - \text{U})$ and inversely as the cube root of V. When T = 12, U = 4 and V = 8.

(i) Determine the equation connecting T, U and V. (3 marks)

$$\text{T} \alpha \frac{(10 - \text{U})^2}{\sqrt[3]{\text{V}}}$$

$$\therefore \text{T} = \frac{k(10 - \text{U})^2}{\sqrt[3]{\text{V}}} \checkmark$$

Given T = 12 when U = 4 and V = 8;

$$12 = \frac{k(10-4)^2}{\sqrt[3]{8}}$$

$$12 = \frac{k \times 6^2}{2}$$

$$36k = 24$$

$$k = \frac{24}{36} = \frac{2}{3} \checkmark$$

$\therefore$ the equation: $\boxed{\mathbf{T} = \dfrac{\frac{2}{3}(\mathbf{10 - U})^2}{\sqrt[3]{\mathbf{V}}}}$ ✓ or $\boxed{\mathbf{T} = \dfrac{\mathbf{2(10 - U)^2}}{\mathbf{3\sqrt[3]{V}}}}$ ✓

(ii) Find U when $\text{T} = 5\frac{2}{5}$ and $\text{V} = 15\frac{5}{8}$. (3 marks)

$$\frac{27}{5} = \frac{2(10 - \text{U})^2}{3 \times \sqrt[3]{15.625}}$$

$$27 \times 3 \times \sqrt[3]{15.625} = 5 \times 2(10 - \text{U})^2$$

$$27 \times 3 \times 2.5 = 10(10 - \text{U})^2 \checkmark$$

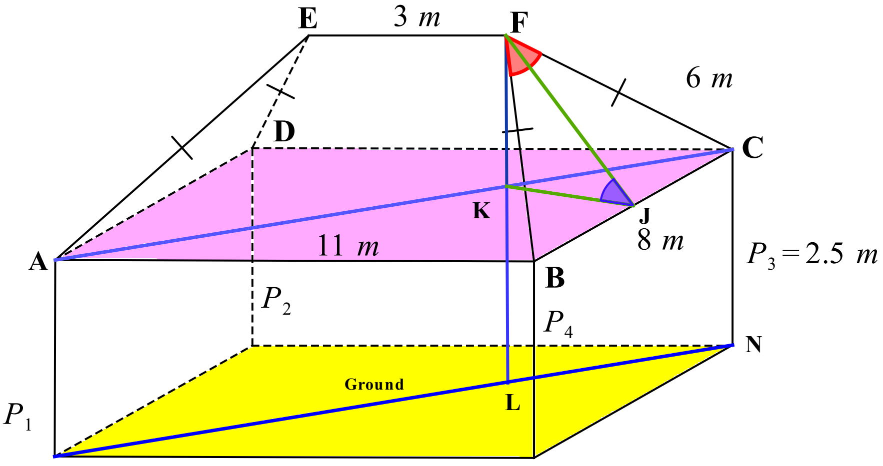
$$\frac{27 \times 3 \times 2.5}{10} = \cancel{10}^1 (10 - \text{U})^2 \times \frac{1}{\cancel{10}^1}$$

$$\sqrt{20.25} = \sqrt{(10 - \text{U})^2} \checkmark$$

$$\pm 4.5 = 10 - \text{U}$$

$\therefore \text{U} = 10 - 4.5 = \mathbf{5.5}$ ✓ Or $\text{U} = 10 + 4.5 = \mathbf{14.5}$ ✓

19. The following figure shows a tent erected on a level ground. The roof ABCDEF of the tent is supported by four vertical posts P_1, P_2, P_3 and P_4 each of height 2.5 m. The ridge EF = 3 m is centrally placed. Further, AB = 11 m, BC = 8 m and FB = FC = ED = EA = 6 m.



Calculate:

(a) the length of the projection of FC on the ground correct to 4 significant figures. (3 marks)

$\text{KJ} = \dfrac{11-3}{2} = 4m$ ✓ | $\text{BJ} = \text{JC} = \dfrac{8}{2} = 4m$

Plane ABCD = Rectangular part of the ground formed by the supporting posts then translated.

The projection of FC on the ground is LN.

But LN = KC

$\text{KC} = \sqrt{\text{KJ}^2 + \text{JC}^2}$

$= \sqrt{4^2 + 4^2}$ ✓

$=$ **5.657 m** ✓

(b) the height of the ridge EF above the ground. (3 marks)

Required length is FL.

$\text{FL} = \text{FK} + \text{KL}$

$= \text{FK} + 2.5m$

Using $\triangle$FKC;

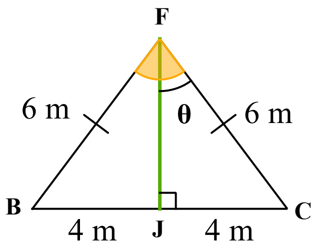
$\text{FK} = \sqrt{\text{FC}^2 - \text{KC}^2} = \sqrt{6^2 - 32}$ ✓

$= 2m$ ✓

$\text{FL} = 2m + 2.5m$

$=$ **4.5 m** ✓

(c) the angle between edge FB and edge FC. (2 marks)



Required angle is $\angle$BFC.

$\sin\theta = \dfrac{4}{6} \Rightarrow \theta = \sin^{-1} 0.6667$

$= 41.81°$ ✓

$\angle \text{BFC} = 2 \times 41.81°$

$=$ **83.62°** ✓

Alternatively, using cosine rule;

$8^2 = 6^2 + 6^2 - (2 \times 6 \times 6 \cos \text{F})$

$\angle \text{F} = \cos^{-1}\left(\dfrac{6^2 + 6^2 - 8^2}{2 \times 6 \times 6}\right)$ ✓

$= \cos^{-1} 0.1111$

$=$ **83.62°** ✓

(d) the angle between the plane FBC and the ground. (2 marks)

Angle between the plane FBC and the ground = angle between the plane FBC and pane ABCD

= angle between FJ and its projection JK on ABCD

Using triangle FKJ; $\tan \text{J} = \dfrac{\text{FK}}{\text{KJ}} \Rightarrow \text{J} = \tan^{-1}\left(\dfrac{\text{FK}}{\text{KJ}}\right) = \tan^{-1}\left(\dfrac{2}{4}\right)$ ✓

$= \tan^{-1} 0.5$

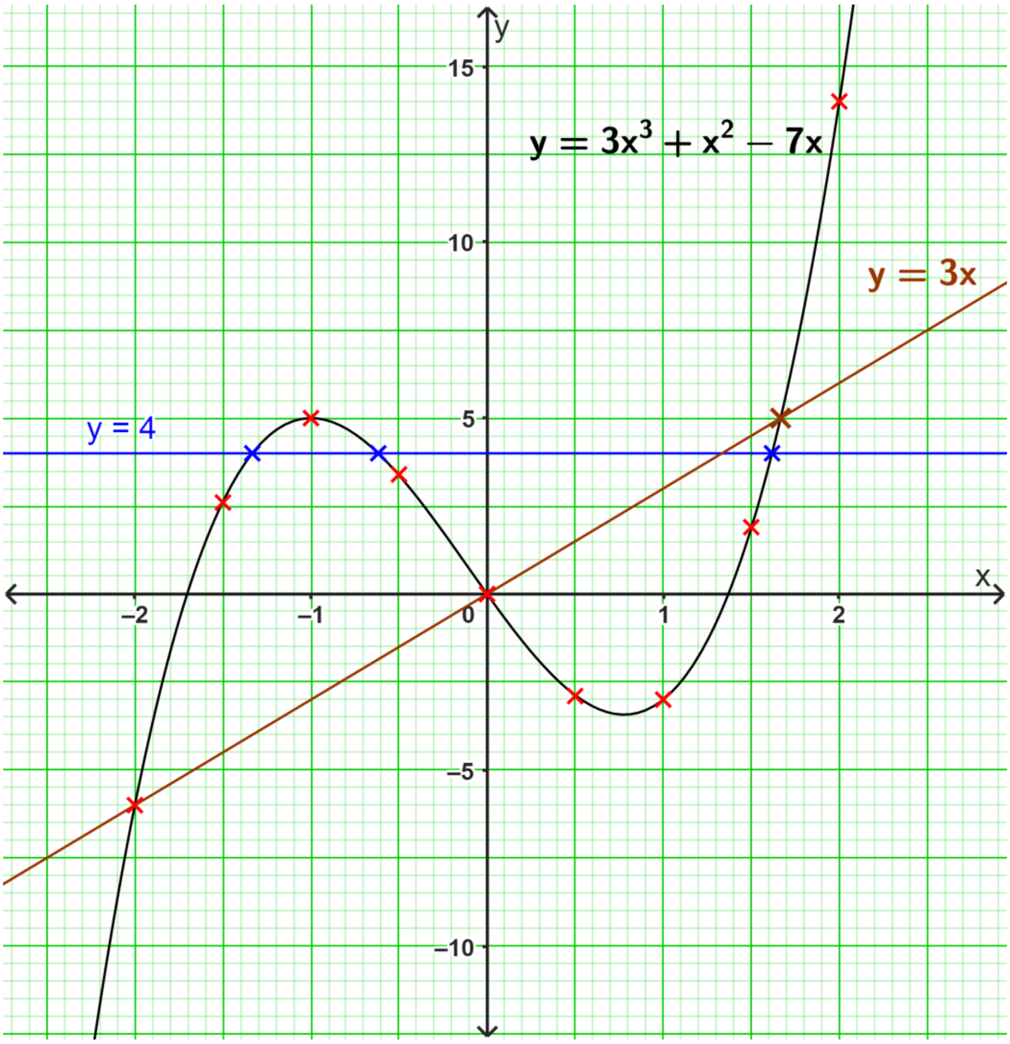
$=$ **26.57°** ✓

20. The table below shows values of x and some values of y for the curve $y = 3x^3 + x^2 - 7x$ in the range $-2 \le x \le 2$.

(a) Complete the table by filling the missing values of y correct to 1 decimal place. (2 marks)

x	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
y	**−6**	2.6	5	**3.4**	**0**	−2.9	**−3**	**1.9**	14

(b) On the grid provided, draw the graph of $y = 3x^3 + x^2 - 7x$ for $-2 \le x \le 2$. Use the scale: 2 cm for 1 unit on the x-axis and 2 cm for 5 units on the y-axis. (3 marks)



(c) Use the graph to solve the equation:

(i) $3x^3 + x^2 - 7x - 4 = 0$ (2 marks)

$$\left.\begin{aligned} y &= 3x^3 + x^2 - 7x \\ 0 &= -4 + 3x^3 + x^2 - 7x \end{aligned}\right\} -$$

$y = 4$ ✓

$x =$ **−1.3, −0.6, 1.6** ✓

(ii) $3x^3 + x^2 - 10x = 0$ (3 marks)

$$\left.\begin{aligned} y &= -7x + x^2 + 3x^3 \\ 0 &= -10x + x^2 + 3x^3 \end{aligned}\right\} -$$

$y = 3x$ ✓

x	0	−2
y	0	−6

$x =$ **−2, 0, 1.7** ✓

21.

(a) Fadhili deposited Ksh. 400 000 in an account that paid compound interest on deposits at a rate of 7% p.a. At the end of 3 years, he withdrew all the money from the account.

(i) Calculate the amount that Fadhili withdrew. (2 marks)

$$\text{Accumulated amount} = 400,000\left(1+\frac{7}{100}\right)^3 ✓$$

$$= 400,000 \times 1.225043$$

$$= \text{Ksh. } 490,017.20 \qquad \therefore \text{ Amount he withdrew is } \mathbf{Ksh.\ 490,017} ✓$$

(ii) Fadhili invested the withdrawn amount in shares. The value of the shares depreciated at a rate of 1.5% every 6 months. Determine the value of the shares at the end of 2 years correct to 2 decimal places. (3 marks)

$$A = P\left(1-\frac{r}{100}\right)^n ;\ n = 2\times 2 = 4 ✓$$

$$\text{Amount} = 490,017.20\left(1-\frac{1.5}{100}\right)^4 ✓$$

$$= 490,017.20 \times 0.94133655$$

$$= \mathbf{Ksh.\ 461,217.10} ✓$$

(iii) Determine the gain or loss from Fadhili's investments in the 5 years. (1 mark)

Negative if loss, positive if gain.

$$\text{Gain/loss} = \text{Amount at the end of 5 years} - \text{Initial amount}$$

$$= \text{Ksh. } (461,217.10 - 400,000)$$

$$= \mathbf{Ksh.\ 61,217.10\,(gain)} ✓$$

(b) Nyambuto invested Ksh. 400 000 in a financial institution that paid compound interest at the rate of 6% per annum. After n years, the amount had accumulated to Ksh. 500 000. Calculate the value of n, correct to the nearest whole number. (4 marks)

$$\text{From } A = P\left(1+\frac{r}{100}\right)^n \Rightarrow \left(1+\frac{r}{100}\right)^n = \frac{A}{P}$$

Taking logs on both sides;

$$n\log\left(1+\frac{r}{100}\right) = \log A - \log P$$

$$\therefore n = \frac{\log A - \log P}{\log\left(1+\frac{r}{100}\right)} ✓$$

$$n = \frac{\log 500000 - \log 400000}{\log\left(1+\frac{6}{100}\right)} ✓$$

$$= \frac{5.69897 - 5.60206}{0.0253059} ✓$$

$$= 3.8295$$

$$\approx \mathbf{4\ years} ✓$$

22. The probabilities of obtaining scores 1, 2, 3, 4 and 5 using a biased pentagonal spinner were recorded as shown in the following table.

Score	1	2	3	4	5
Probability	k	0.1	0.25	$2k$	0.2

(a) Determine:

(i) the value of k. (2 marks)

$$k+0.1+0.25+2k+0.2=1 \checkmark$$
$$3k+0.55=1$$
$$3k=1-0.55$$
$$3k=0.45 \Rightarrow k=\frac{0.45}{3}=0.15 \checkmark$$

(ii) the probability of obtaining a score of 4. (1 mark)

$$P(score=4)=2k$$
$$=2\times 0.15=0.3 \checkmark$$

(b) The spinner was span twice.

(i) Work out the probability of obtaining an even number in the first spin and an odd number in the second spin. (4 marks)

	First spin				
Second spin	**1**	**2**	**3**	**4**	**5**
1	(1,1)	**(2,1)**	(3,1)	**(4,1)**	(5,1)
2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)
3	(1,3)	**(2,3)**	(3,3)	**(4,3)**	(5,3)
4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)
5	(1,5)	**(2,5)**	(3,5)	**(4,5)**	(5,5)

✓✓ 2 marks

Probability space = 25
Favourable outcomes = 6 } ✓

$$P(even\ and\ odd)=\frac{6}{25} \text{ or } 0.24 \checkmark$$

Alternatively:

$$P(2,1) \textbf{ OR } P(2,3) \textbf{ OR } P(2,5) \textbf{ OR } P(4,1) \textbf{ OR } P(4,3) \textbf{ OR } P(4,5) \checkmark$$

$$P(2,1)+P(2,3)+P(2,5)+P(4,1)+P(4,3)+P(4,5)$$

$$(0.1\times 0.15)+(0.1\times 0.25)+(0.1\times 0.2)+(0.3\times 0.15)+(0.3\times 0.25)+(0.3\times 0.2) \checkmark$$

$$0.015+0.025+0.02+0.045+0.075+0.06 \checkmark$$

0.24✓

(ii) The total score, **S**, for the two spins was obtained. Determine the probability that $S\geq 9$. (3 marks)

$$P(4,5) \textbf{ OR } P(5,4) \textbf{ OR } P(5,5) \checkmark$$

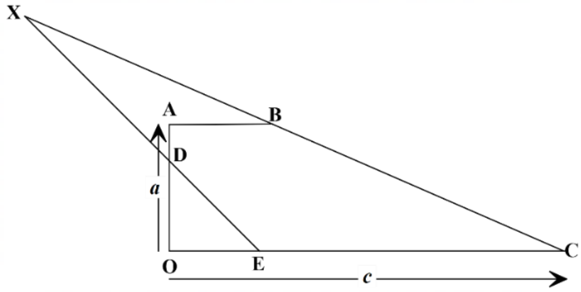
$$P(4,5)+P(5,4)+P(5,5)$$

$$(0.3\times 0.2)+(0.2\times 0.3)+(0.2\times 0.2) \checkmark$$

$$0.06+0.06+0.04$$

0.16✓

23. In the figure below, OABC is a trapezium. **AB** is parallel to **OC** and **OC** = 4 **AB**. D is point on **OA** such that OD : DA = 3: 1 and E is a point on **OC** such that OE : EC = 1 : 3.



(a) Given that **OC** = **c** and **OA** = **a**, express in terms of **a** and **c**:

(i) **ED.** (1 mark)

$$\mathbf{ED} = \mathbf{OD} - \mathbf{OE}$$
$$= \tfrac{3}{4}\mathbf{OA} - \tfrac{1}{4}\mathbf{OC}$$
$$= \boxed{\tfrac{3}{4}\mathbf{a} - \tfrac{1}{4}\mathbf{c}} \text{ or } \boxed{\tfrac{1}{4}(3\mathbf{a} - \mathbf{c})} ✓$$

(ii) **CB.** (1 mark)

$$\mathbf{CB} = \mathbf{OB} - \mathbf{OC}$$
$$= \mathbf{OA} + \mathbf{AB} - \mathbf{OC}$$
$$= \mathbf{OA} + \tfrac{1}{4}\mathbf{OC} - \mathbf{OC}$$
$$= \mathbf{a} + \tfrac{1}{4}\mathbf{c} - \mathbf{c}$$
$$= \boxed{\mathbf{a} - \tfrac{3}{4}\mathbf{c}} \text{ or } \boxed{\tfrac{1}{4}(4\mathbf{a} - 3\mathbf{c})} ✓$$

(b) Line ED and CB produced intersect at X such that **EX** = *h***ED** and **CX** = *k***CB**, where *h* and *k* are scalars.

(i) Express **EX** in terms of **a**, **c** and *h*. (1 mark)

$$\mathbf{EX} = h\left(\frac{3}{4}\mathbf{a} - \frac{1}{4}\mathbf{c}\right) = \boxed{\frac{3}{4}h\mathbf{a} - \frac{1}{4}h\mathbf{c}} \text{ or } \boxed{\frac{1}{4}h(3\mathbf{a} - \mathbf{c})} ✓$$

(ii) Express **CX** in terms of **a**, **c** and *k*. (1 mark)

$$\mathbf{CX} = k\left(\mathbf{a} - \frac{3}{4}\mathbf{c}\right) = \boxed{k\mathbf{a} - \frac{3}{4}k\mathbf{c}} \text{ or } \boxed{\frac{1}{4}k(4\mathbf{a} - 3\mathbf{c})} ✓$$

(c) Determine the values of *h* and *k*. (5 marks)

Expressing **OX** in 2 ways:

$$\mathbf{OX} = \mathbf{OE} + \mathbf{EX}$$
$$= \frac{1}{4}\mathbf{c} + \frac{3}{4}h\mathbf{a} - \frac{1}{4}h\mathbf{c}$$
$$= \frac{3}{4}h\mathbf{a} + \left(\frac{1}{4} - \frac{1}{4}h\right)\mathbf{c} \ldots\ldots(i) ✓$$
$$\mathbf{OX} = \mathbf{OC} + \mathbf{CX}$$
$$= \mathbf{c} + k\mathbf{a} - \frac{3}{4}k\mathbf{c}$$
$$= k\mathbf{a} + \left(1 - \frac{3}{4}k\right)\mathbf{c} \ldots\ldots(ii) ✓$$

$$\boxed{k = \frac{3}{4}h} \text{ and } \boxed{1 - \frac{3}{4}k = \frac{1}{4} - \frac{1}{4}h}$$

Substituting for *k* in ↑

$$1 - \frac{3}{4}\left(\frac{3}{4}h\right) = \frac{1}{4} - \frac{1}{4}h ✓$$
$$1 - \frac{9}{16}h = \frac{1}{4} - \frac{1}{4}h$$
$$\frac{5}{16}h = \frac{3}{4} \Rightarrow h = \frac{3}{4} \times \frac{16}{5} = \mathbf{\frac{12}{5}} ✓$$
$$k = \frac{3}{4} \times \frac{12}{5} = \mathbf{\frac{9}{5}} ✓$$

Alternatively, express **CX** or **EX** in 2 ways…

$$\left\{\begin{array}{l} \mathbf{CX} = \mathbf{CE} + \mathbf{EX} \ldots\ldots(i) \\ \quad = -\tfrac{1}{4}(3 + h)\mathbf{c} + \tfrac{3}{4}h\mathbf{a} \\ \mathbf{CX} = k\mathbf{a} - \tfrac{3}{4}k\mathbf{c} \ldots\ldots(ii) \end{array}\right\}$$

$$\left\{\begin{array}{l} \mathbf{EX} = \mathbf{EC} + \mathbf{CX} \\ \quad = k\mathbf{a} + \left(\tfrac{3}{4} - \tfrac{3}{4}k\right)\mathbf{c} \ldots\ldots(i) \\ \mathbf{EX} = \tfrac{3}{4}h\mathbf{a} - \tfrac{1}{4}h\mathbf{c} \ldots\ldots(ii) \end{array}\right\}$$

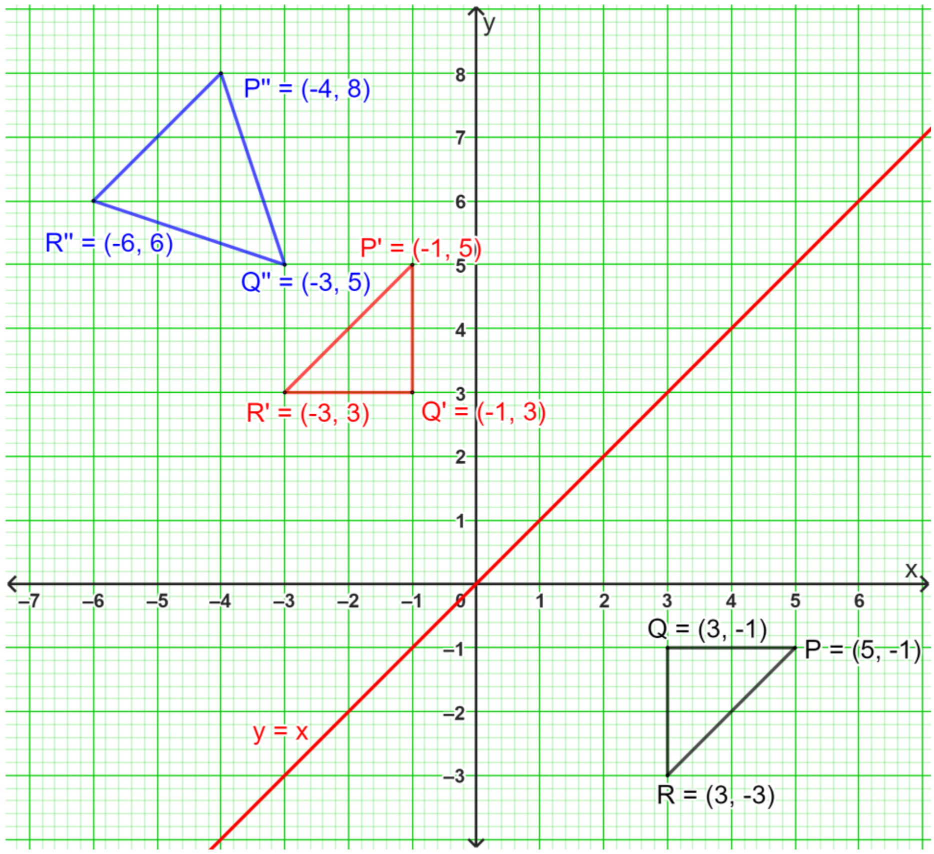
(d) Determine the ratio ED : DX. (1 mark)

$$\mathbf{DX} = \frac{7}{20}(3\mathbf{a} - \mathbf{c}) \text{ and } \mathbf{ED} = \frac{1}{4}(3\mathbf{a} - \mathbf{c}) \Rightarrow \frac{\text{ED}}{\text{DX}} = \frac{1}{4} \times \frac{20}{7} = \frac{5}{7}$$

$$\therefore \text{ED} : \text{DX} = \mathbf{5 : 7} ✓$$

24. PQR is a triangle with vertices $P(5,-1)$, $Q(3,-1)$ and $R(3,-3)$.

(a) On the grid provided, draw triangle PQR. (1 mark)



(b) On the same grid, draw $\Delta P'Q'R'$ the image of ΔPQR under a reflection in the line $y = x$. (3 marks)

$$X(a,b)\xrightarrow[y=x]{\text{Reflect}}X'(b,a)\text{, use the matrix }\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\text{ or construct.}$$

$$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\overset{\quad P\quad Q\quad R}{\begin{pmatrix}5 & 3 & 3\\ -1 & -1 & -3\end{pmatrix}}=\begin{pmatrix}(0\times5)+(1\times-1) & (0\times3)+(1\times-1) & (0\times3)+(1\times-3)\\ (1\times5)+(0\times-1) & (1\times3)+(0\times-1) & (1\times3)+(0\times-3)\end{pmatrix}✓=\overset{\quad P'\quad Q'\quad R'}{\begin{pmatrix}-1 & -1 & -3\\ 5 & 3 & 3\end{pmatrix}}$$

$\therefore$ $\mathbf{P'(-1,5), Q'(-1,3), R'(-3,3)}$ ✓

(c) $\Delta P''Q''R''$ is the image of $\Delta P'Q'R'$ under a transformation matrix $\mathbf{T}=\begin{pmatrix}1.5 & -0.5\\ -0.5 & 1.5\end{pmatrix}$.

(i) Find the coordinates of $P''Q''R''$. (2 marks)

$$\begin{pmatrix}1.5 & -0.5\\ -0.5 & 1.5\end{pmatrix}\overset{\quad P'\quad Q'\quad R'}{\begin{pmatrix}-1 & -1 & -3\\ 5 & 3 & 3\end{pmatrix}}=\begin{pmatrix}(1.5\times-1)+(-0.5\times5) & (1.5\times-1)+(-0.5\times3) & (1.5\times-3)+(-0.5\times3)\\ (-0.5\times-1)+(1.5\times5) & (-0.5\times-2)+(1.5\times3) & (-0.5\times-3)+(1.5\times3)\end{pmatrix}✓$$

$$=\overset{\quad P''\quad Q''\quad R''}{\begin{pmatrix}-4 & -3 & -6\\ 8 & 5 & 6\end{pmatrix}}$$

$\therefore$ $\mathbf{P''(-4,8), Q''(-3,5), R''(-6,6)}$ ✓

(ii) On the same grid, draw triangle $P''Q''R''$. (1 mark)

(d) Determine a single transformation matrix that maps ΔPQR directly to $\Delta P''Q''R''$. (3 marks)

$$\begin{pmatrix}1.5 & -0.5\\ -0.5 & 1.5\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}✓=\begin{pmatrix}(1.5\times0)+(-0.5\times1) & (1.5\times1)+(-0.5\times0)\\ (-0.5\times0)+(1.5\times1) & (-0.5\times1)+(1.5\times0)\end{pmatrix}✓=\begin{pmatrix}\mathbf{-0.5} & \mathbf{1.5}\\ \mathbf{1.5} & \mathbf{-0.5}\end{pmatrix}✓$$