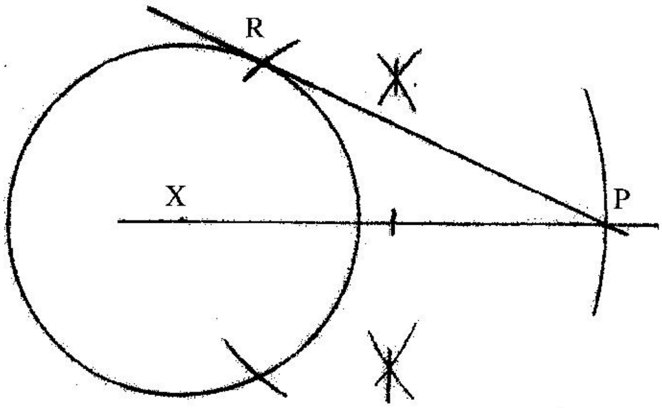
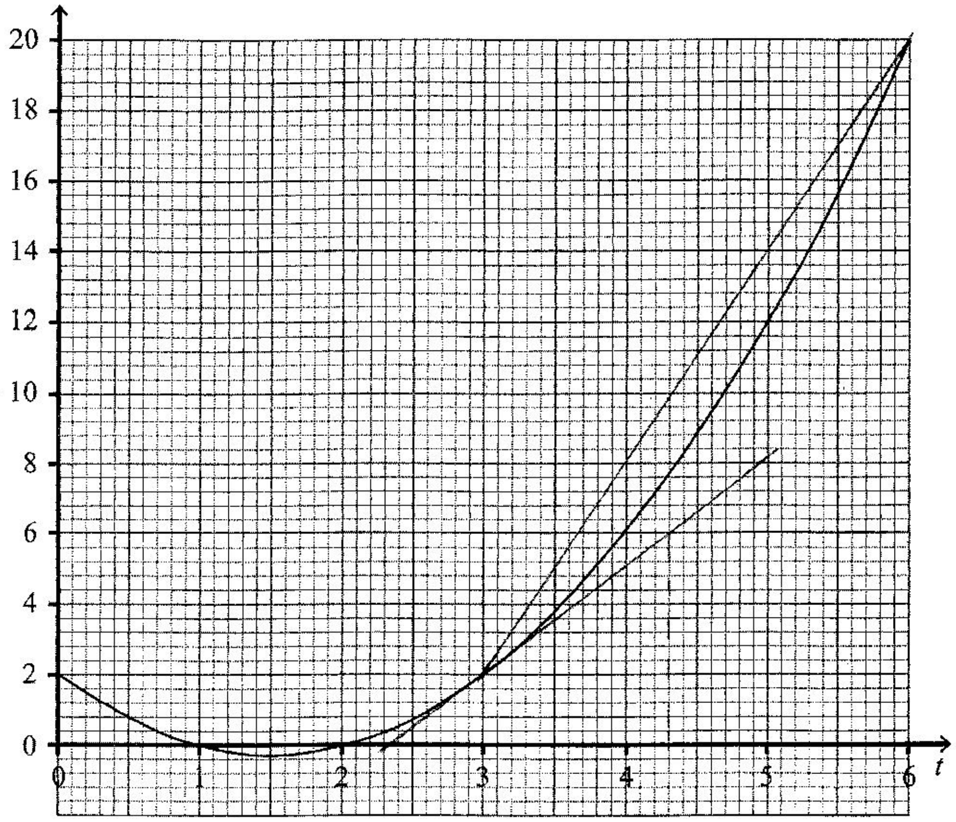
5.1.2 Mathematics Alternative A Paper 2 (121/2)

1.	$\dfrac{5\log 4 - 4\log 5}{\frac{1}{5}\log 4 + \frac{1}{4}\log 5}$ $= \dfrac{3.010299957 - 2.795880017}{0.120411998 + 0.174742501}$ $= 0.726466785$ $\simeq 07265$ (4 s.f.)	M1 A1 2	
2.	$\left(\dfrac{r}{p}\right)^2 = \dfrac{m^2}{n-1}$ $n - 1 = \left(\dfrac{mp}{r}\right)^2$ $n = \left(\dfrac{mp}{r}\right)^2 + 1$	M1 M1 A1 3	squaring
3.	Fraction filled by inlet tap in $1h = \dfrac{1}{6}$ Fraction filled when two taps open in $1h = \dfrac{1}{10}$ $\therefore$ fraction emptied by outlet tap in $1h = \dfrac{1}{6} - \dfrac{1}{10}$ $= \dfrac{1}{15}$ Time for outlet tap to empty tank = 15h	B1 M1 A1 3	for $\dfrac{1}{6}$ or $\dfrac{1}{10}$
4.	$\underset{\sim}{R} = 6\underset{\sim}{i} - 9\underset{\sim}{j} + 3\underset{\sim}{k} + 6\underset{\sim}{i} - 8\underset{\sim}{j} - 6\underset{\sim}{k}$ $= 12\underset{\sim}{i} - 17\underset{\sim}{j} - 3\underset{\sim}{k}$ $\lvert R \rvert = \sqrt{12^2 + 17^2 + 3^2}$ $= \sqrt{442}$ $= 21.02 \simeq 21$ (2 s.f.)	B1 M1 A1 3	
5.	$\text{Sin}\,(2t + 10)° = 0.5$ $2t + 10 = 30°, 150°$ $t = 10°, 70°$	B1 B1 2	

6.			
	Drawing circle Fixing point P Bisecting XP and drawing tangent RP = $5.4 \pm 0.1cm$	B1 B1 B1 B1	
		4	
7.	Amount for Kago $= 30000 + \frac{12}{100} \times 30000 \times 5$ $= 48000$	 B1	
	Compound interest rate for Nekesa $30000\left(1 + \frac{r}{100}\right)^5 = 48000$ $\left(1 + \frac{r}{100}\right)^5 = \frac{48000}{30000} = 1.6$ $1 + \frac{r}{100} = \sqrt[5]{1.6}$ $r = 100(1.098560543 - 1)$ $= 9.9\%$	 M1 M1 A1	
		4	
8.	Differences from assumed mean $-6 - 2 + 0 + 2 + 3 + 6 + 9 - 5 + 6 + 3 + 9$ $-2 + 3 - 6 - 2 + 3 + 2 + 0 + 6 + 9 = 38$	 M1	differences from the assumed mean
	$\therefore mean = 96 + \frac{38}{20}$ $= 97.9$	M1 A1	
		3	

9.	$x + y = 17$..........(i) $xy - 5x = 32$......(ii) from (i) $y = 17 - x$ substituting $y = 17 - x$ in (ii) $x(17 - x) - 5x = 32$ $17x - x^2 - 5x = 32$ $x^2 - 12x + 32 = 0$ $(x - 4)(x - 8) = 0$ $x = 4$ *or* $x = 8$ substituting x = 4 in (i) $4 + y = 17 \Rightarrow y = 13$ substituting x = 8 in (ii) $8 + y = 17 \Rightarrow y = 9$	M1 M1 A1 B1 4	for substitution or elimination for both 4 and 8 for both 13 and 9
10	$\frac{\sqrt{5}}{\sqrt{5} - 2} = \frac{\sqrt{5}}{\sqrt{5} - 2} \times \frac{\sqrt{5} + 2}{\sqrt{5} + 2}$ $= \frac{5 + 2\sqrt{5}}{5 - 4}$ $= 5 + 2\sqrt{5}$	M1 A1 2	
11.	minimum possible area $= \frac{1}{2}(6.35 \times 3.45)$ $= 10.95375 \text{ cm}^2$ maximum possible area $= \frac{1}{2} \times 6.45 \times 3.55$ $= 11.44875 \text{ cm}^2$ maximum absolute error in area $= \frac{11.44875 - 10.95375}{2}$ $= 0.2475 \text{ cm}^2$	M1 M1 A1 3	for both expressions - min. and max. areas
12.	(a) $(1 + x)^7 = 1^7 + 7 \times 1^6 \times x + 21 \times 1^5 \times x^2 + 35 \times 1^4 \times x^3 + \ldots$ $= 1 + 7x + 21x^2 + 35x^3$ (b) $(0.94)^7 = [1 + (-0.06)]^7$ $= 1 + 7 \times (-0.06) + 21 \times (-0.06)^2 + 35 \times (-0.06)^3$ $= 1 - 0.42 + 0.0756 - 0.00756$ $= 0.64804$	B1 M1 A1 3	

13

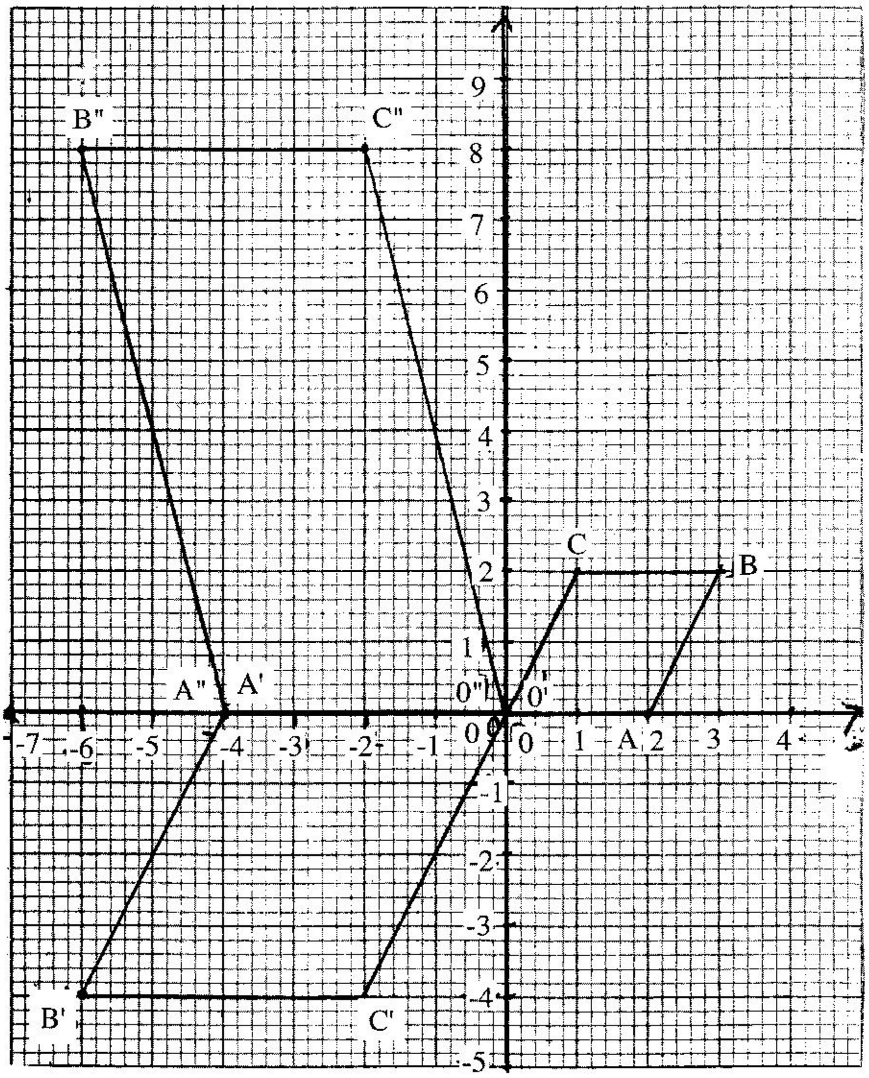


(a)	Average rate of change between t = 3 and t = 6 $\frac{20-2}{6-3}$ $= \frac{18}{3} = 6$	M1 A1	
(b)	Gradient at t = 3 seconds $\frac{6-0}{4.3-2.3} = \frac{6}{2}$ $= 3 \pm 0.1$	M1 A1	or equivalent
		4	

14.	(a) Let UV be x cm: $VT \times UT = ST^2$ $(x+8)8 = 12^2$ $8x = 144 - 64$ $= 80$ $x = 10$ cm (b) $VX = \frac{2}{5} \times 10 = 4$ cm $XU = 10 - 4 = 6$ cm $SX \times XW = VX \times XU$ $SX \times 3 = 4 \times 6$ $SX = 8$ cm	M1 A1 M1 A1 4	
15.	$P \propto \frac{Q}{\sqrt{R}} \Rightarrow P = \frac{kQ}{\sqrt{R}}$ $8 = \frac{k \times 10}{\sqrt{16}}$ $k = 3.2$ $P = \frac{3.2Q}{\sqrt{R}}$	M1 A1 B1 3	
16.	$OC = \frac{\sqrt{24^2 + 10^2}}{2}$ $= 13$ $\angle VCO = \cos^{-1}\frac{13}{26}$ $= 60°$	M1 M1 A1 3	

17.	(a) (i) $180000 + (11 - 1)x = 288000$ $10x = 108000$ $x = 10800$	M1 A1	
	(a) (ii) $S_{11} = \frac{11}{2}(180000 + 288000)$ $= 2574000$	M1 A1	
	(b) $\frac{150000 \times 1.1^{10}}{12}$ $= 32422$	M1 A1	
	(c) (i) $\frac{[150000 \times (1.1^{11} - 1)]}{(1.1 - 1)}$ $= 2779675$	M1 A1	
	(c) (ii) Difference between monthly averages for the 11 years $\frac{2779675 - 2574000}{11 \times 12}$ $= 1558$	M1 A1	
		10	
18.	(a) $\begin{array}{cc} & \begin{array}{cccc} O & A & B & C \end{array} \\ \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} & \begin{pmatrix} 0 & 2 & 3 & 1 \\ 0 & 0 & 2 & 2 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} O' & A' & B' & C' \end{array} \\ \begin{pmatrix} 0 & -4 & -6 & -2 \\ 0 & 0 & -4 & -4 \end{pmatrix} \end{array}$ co-ordinates of O'A'B'C' O' (0, 0), A' (-4, 0), B' (-6, -4), C' (-2, -4)	M1 A1	

18. continued



	B1	OABC √ drawn
	B1	O'A'B'C' √ drawn
	B1	O"A"B"C" √ drawn

(b)

$$\begin{pmatrix}1&0\\0&-2\end{pmatrix}\begin{pmatrix}0&-4&-6&-2\\0&0&-4&-4\end{pmatrix}=\begin{array}{c}\begin{array}{cccc}O'' & A'' & B'' & C''\end{array}\\ \begin{pmatrix}0&-4&-6&-2\\0&0&8&8\end{pmatrix}\end{array}$$

M1

A1

(c)

$$\begin{pmatrix}1&0\\0&-2\end{pmatrix}\begin{pmatrix}-2&0\\0&-2\end{pmatrix}=\begin{pmatrix}-2&0\\0&4\end{pmatrix}$$

M1 or equivalent

inverse $\frac{1}{-8}\begin{pmatrix}4&0\\0&-2\end{pmatrix}$

M1

$=\begin{pmatrix}-\frac{1}{2}&0\\0&\frac{1}{4}\end{pmatrix}$

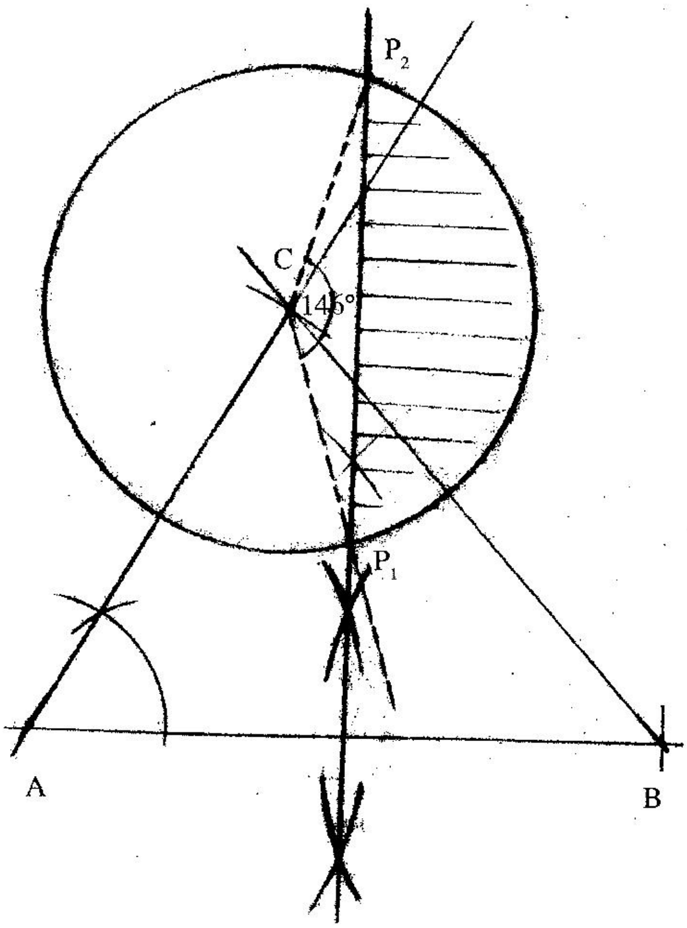
A1

10

19.	(a) (i) $\underset{\sim}{PN} = \frac{5}{6}\underset{\sim}{q} - \underset{\sim}{p}$	B1	
	(ii) $\underset{\sim}{QM} = \frac{2}{5}\underset{\sim}{p} - \underset{\sim}{q}$	B1	
	(b) (i) $\underset{\sim}{OX} = \underset{\sim}{p} + k\left(\frac{5}{6}\underset{\sim}{q} - \underset{\sim}{p}\right)$	B1	
	$\underset{\sim}{OX} = \underset{\sim}{q} + r\left(\frac{2}{5}\underset{\sim}{p} - \underset{\sim}{q}\right)$	B1	
	(ii) $\underset{\sim}{p} + k\left(\frac{5}{6}\underset{\sim}{q} - \underset{\sim}{p}\right) = \underset{\sim}{q} + r\left(\frac{2}{5}\underset{\sim}{p} - \underset{\sim}{q}\right)$	M1	
	$\underset{\sim}{p}(1-k) + \frac{5}{6}k\underset{\sim}{q} = \underset{\sim}{q}(1-r) + \frac{2}{5}r\underset{\sim}{p}$		
	$1 - k = \frac{2}{5}r$ and $1 - r = \frac{5}{6}k$	M1	
	$1 - r = \frac{5}{6}\left(1 - \frac{2}{5}r\right)$	M1	
	$1 - r = \frac{5}{6} - \frac{1}{3}r$		
	$\frac{1}{6} = \frac{2}{3}r \Rightarrow r = \frac{1}{4}$		
	$k = 1 - \frac{2}{5}r \Rightarrow k = 1 - \frac{2}{5} \times \frac{1}{4} = \frac{9}{10}$	A1	for both values of **r** and **k**
	(iii) $\underset{\sim}{QX} = \frac{1}{4}\underset{\sim}{QM}$	M1	
	$\underset{\sim}{MX} = \frac{3}{4}\underset{\sim}{QM}$		
	$\therefore \underset{\sim}{MX} : \underset{\sim}{XQ} = \frac{3}{4} : \frac{1}{4} = 3 : 1$	A1	
		10	

20.	(a) (i) July basic salary $= 17000 \times 1.02$ $= 17340$	 M1 A1	
	(ii) Total taxable income $= 17340 + 6000 + 2500 + 1800$ $= 27640$	 M1 A1	
	(b) Gross tax		
	1st bracket: $9680 \times 10\% = 968$	M1	
	2nd bracket: $(18800 - 9680) \times 15\% = 1368$	M1	
	3rd bracket: $(27640 - 18800) \times 20\% = 1768$	M1	[27649 - (9680 + 9120)]20%
	Gross tax: $968 + 1368 + 1768$ $= 4104$	M1 A1	
	Net tax: $4104 - 1056 = 3048$	B1	
		10	

21.

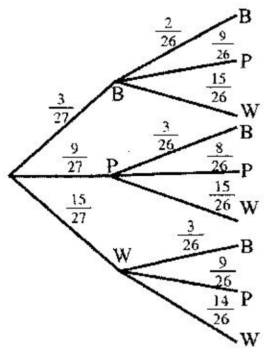


B1 construction of 60°

B1 completion of Δ

(a) locus of P	B1	
locus of Q	B1	
(b) (i) shading region R	B2	
(ii) area of shaded region area of minor sector P_1CP_2 $= \frac{146}{360} \times \pi \times 3.5^2$ $\simeq 15.6\ cm^2$	M1	($\angle P_1CP_2 = 146° \pm 1°$)
area of ΔP_1CP_2 $\frac{1}{2} \times 3.5^2 \sin 146°$ $\simeq 3.4\ cm^2$	M1	
$\therefore$ shaded area 15.6 - 3.4	M1	
=12.2 cm^2	A1	
	10	

22.	(a) distance from T to U $= 2 \times 6370 \times \frac{22}{7} \times \frac{12}{360}$	M1	
	speed $= \dfrac{2 \times 6370 \times \frac{22}{7} \times \frac{12}{360}}{1\frac{1}{3}}$	M1	
	$= 1001$ km/h	A1	
	(b) time $= \dfrac{2 \times 6370 \times \frac{22}{7} \times \frac{30}{360} \cos 9^\circ}{1001 \times \frac{90}{100}}$	M1 M1	
	$= 3.658104965$ h $\simeq 3$ h 39 min	A1	
	(c) Arrival time at U 0700 + 1h 20 min = 0820 h		
	Departure time at U 0820 + 30 min = 0850 h	M1	
	Time difference between U and V $\frac{35-5}{360} \times 24$ = 2h	M1	or equivalent
	Arrival time at V (local time) 0850h + 3h 39min - 2h = 1029h	M1 A1	
		10	

23.	(a) (i) P (brown) $= \frac{3}{27}$	B1	
	(ii) P(pink or white) $= \frac{9}{27} + \frac{15}{27}$	M1	
	$= \frac{8}{9}$	A1	
	(b) (i) P(white and brown) $= \frac{15}{27} \times \frac{3}{26} + \frac{3}{27} \times \frac{15}{26}$	M1 M1	
	$= \frac{5}{78} + \frac{5}{78} = \frac{5}{39}$	A1	
	(ii) white, white + pink, pink + brown, brown $= \frac{15}{27} \times \frac{14}{26} + \frac{9}{27} \times \frac{8}{26} + \frac{3}{27} \times \frac{2}{26}$	M1 M1 M1	
	$= \frac{35}{117} + \frac{4}{39} + \frac{1}{117} = \frac{16}{39}$	A1	
		10	
24.	(a) (i) $\frac{dv}{dt} = 4 - t$ $V = \int (4 - t)dt$ $= 4t - \frac{1}{2}t^2 + c$	B1	
	when t = 0, v = 3 m/s $\therefore 3 = 4 \times 0 - \frac{1}{2} \times 0^2 + c$ $3 = c$	B1	
	$\therefore V = 4t - \frac{1}{2}t^2 + 3$	B1	
	(ii) when t = 2 seconds $V = 4 \times 2 - \frac{1}{2} \times 2^2 + 3$	M1	
	$= 8 - 2 + 3$ $= 9\, m/s$	A1	
	(b) (i) At maximum velocity $\frac{dv}{dt} = 0$ i.e. 4 - t = 0	M1	
	t = 4 seconds	A1	
	(ii) $\int_0^4 4t - \frac{1}{2}t^2 + 3 = \frac{4}{2}t^2 - \frac{1}{2} \times \frac{1}{3}t^3 + 3t\Big]_0^4$	M1	
	$= 2t^2 - \frac{1}{6}t^3 + 3t\Big]_0^4$ $= [2 \times 16 - \frac{1}{6} \times 64 + 12] - 0$	M1	
	$= 32 - 10\frac{2}{3} + 12 = 33\frac{1}{3}$	A1	
		10	