1035

121/2 MS
MATHEMATICS ALT. A
Paper 2
March 2021
MARKING SCHEME

THE KENYA NATIONAL EXAMINATINOS COUNCIL

Kenya Certificate of Secondary Education

MATHEMATICS Alt. A
Paper 2

MARKING SCHEME
(CONFIDENTIAL)

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This marking scheme consists of 16 printed pages.

Turnover

121/2 MATHEMATICS ALT. A

SECTION I

No.	Marking scheme	Marks	Comments
1.	Let the ratio of maize to millet $= x : y$		
	$\frac{60x + 90y}{x + y} = 85$ ✓ — — —	M1	Or equivalent
	$60x + 90y = 85x + 85y$		
	$25x = 5y$ $60x + 90(1-x) = 85$		
			~~or equivalent~~
	$\frac{x}{y} = \frac{1}{5}$		
	$x : y = 1 : 5$	~~A1~~	
	% of maize flour $= \frac{1}{6} \times 100\%$ ✓	M1	$5/30 \times 100$
	$= 16\frac{2}{3}\%$ ✓ ✓	A1 ~~B1~~	accept 16.67
	ratio	3	
2.	Let the first term be a and the common ~~difference~~ r		
	$a + ar = 20 \ldots\ldots (i)$ $ar + ar^2 = 30 \ldots\ldots (ii)$	M1	For both equation correctly formed
	from (i), $a = \frac{20}{1 + r}$		
	from (ii) $a = \frac{30}{r + r^2}$		
	$\frac{20}{1 + r} = \frac{30}{r + r^2}$	M1 ~~M1~~	Equation in one variable
	$2r^2 + 2r = 3 + 3r$		
	$2r^2 - r - 3 = 0$		
	$(r + 1)(2r - 3) = 0$	M1	Correct attempt to solve.
	$r = \frac{3}{2} = 1.5 = 1\frac{1}{2}$.	A1	Accept $1\frac{1}{2}$ or 1.5
		4	

$ar + ar^2 = 20$

$ar + ar^2 = 30$.

$20r - 30 = 0$.

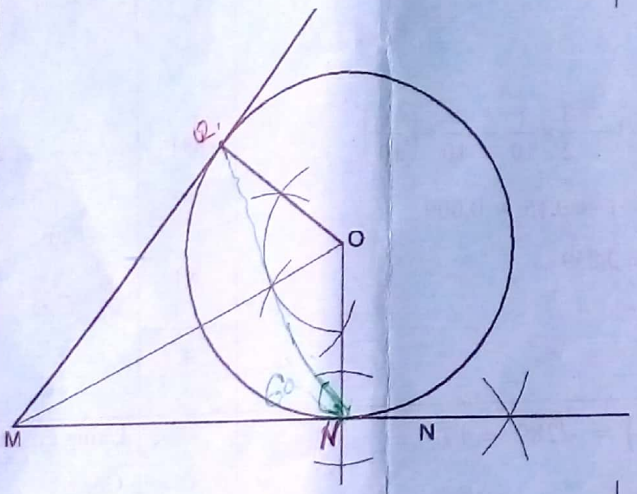
No.	Marking scheme	Marks	Comments
3.	$\frac{1}{\sin 75^\circ} = \frac{4}{\sqrt{6}+\sqrt{2}}$ $= \frac{4(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})}$ $= \frac{4(\sqrt{6}-\sqrt{2})}{6-2}$ ——→ $= \sqrt{6}-\sqrt{2}$	M1 A1 2	Denominator rationalized
4.	(a) $\left(1-\frac{3}{10}x\right)^5 = 1+5\times1\times\left(\frac{-3x}{10}\right)+10\times1\times\left(\frac{-3x}{10}\right)^2$ $+10\times1\times\left(\frac{-3x}{10}\right)^3+5\times1\times\left(\frac{-3x}{10}\right)^4+\left(\frac{-3x}{10}\right)^5$ ✓ $= 1-\frac{3}{2}x+\frac{9}{10}x^2-\frac{27x^3}{100}+\frac{81x^4}{2000}-\frac{243x^5}{100000}$ ——→ (b) ~~When~~ $x = 0.1$, $\left(1-\frac{3}{10}\times\frac{1}{10}\right)^5 \approx 1-\frac{3}{2}\times\frac{1}{10}+\frac{9}{10}\times\left(\frac{1}{10}\right)^2$ - - - $(0.97)^5 = 1-0.15+0.009$ ≈ 0.859	M1 A1 M1 A1 4	Initial expansion correct. Coefficients must be fraction. ✓ Substitution in the 1st three terms. Accept $\frac{859}{1000}$.
5.	$AC = \sqrt{(15^2+8^2)} = \sqrt{289} = 17$ $OC = 8.5$ $OF = \sqrt{5^2+8.5^2} = \sqrt{97.25}$ - - - ——→ $= 9.862$ $\sin\frac{1}{2}\theta = \frac{7.5}{9.862}$ $\frac{1}{2}\theta = \sin^{-1}0.7605$ $\theta = 2\sin^{-1}0.7605$ $\theta = 2\times49.51$ - - - - - - - - - - ——→ $= 99.02^\circ$ (1) - - - - - - - - - - ——→	M1 M1 A1 3	Using cosine rule $\cos\theta = \frac{2\times9.862^2-15^2}{2\times9.862^2}$ 2nd M1 $= -0.1567$ $\theta = 99.01$ A1 $\cos\theta = \frac{2\times9.862^2-15^2}{2\times9.862^2}$. last method else all other method enhanced

$\tan\alpha = \frac{7.5}{6.403} = 1.171$

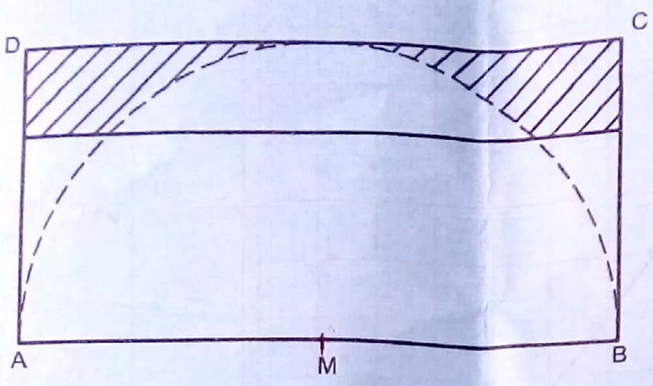
$\alpha = \tan^{-1}1.171$

$\alpha = 49.5\times2$

$= 99$

No.	Marking scheme	Marks	Comments
6.	$y = kx^n$ $320 = k \times 16^n$ (i) $2560 = k \times 64^n$ (ii) $\frac{320}{16^n} = \frac{2560}{64^n} \Rightarrow \frac{1}{2^{4n}} = \frac{8}{2^{6n}}$ . . . . . P $\frac{2^{6n}}{2^{4n}} = 2^3$ $2^{2n} = 2^3$ $2n = 3$ ——————→ $n = \frac{3}{2} = 1.5$ Since n is an index and is easier to read.	M1 M1 A1 3	$\log 16 = n \log 4$ $n = \frac{-3 \log 2}{-2 \log 2}$ $= 1.8$ eliminates one equation in m Equating the indices Accept 3/2 or 1½.
7.	(a)  (b) 3(5x−4) = 2⁷.	√B1 B1 B1 3	Tangent at N √ly constructed Construct 60°N. √ location of the pt of contact on the 2nd tangent Radius at an ∠ of 120° to ON constructed Tangent intersecting √MN at 60° constructed
8.	$15x - 12 = 2^7$ ——————→ ~~$3(5x-4) = 2^7$~~ $15x - 12 = 128$ $15x = 140$ — — — — — — — — — $x = 9\frac{1}{3}$ — — — — — — → 9.333 Accept.	M1 M1 A1 3	√ dropping of loss. Simplify to single term in either side.

$A^n = 8$
$2^{2n} = 2^3$.

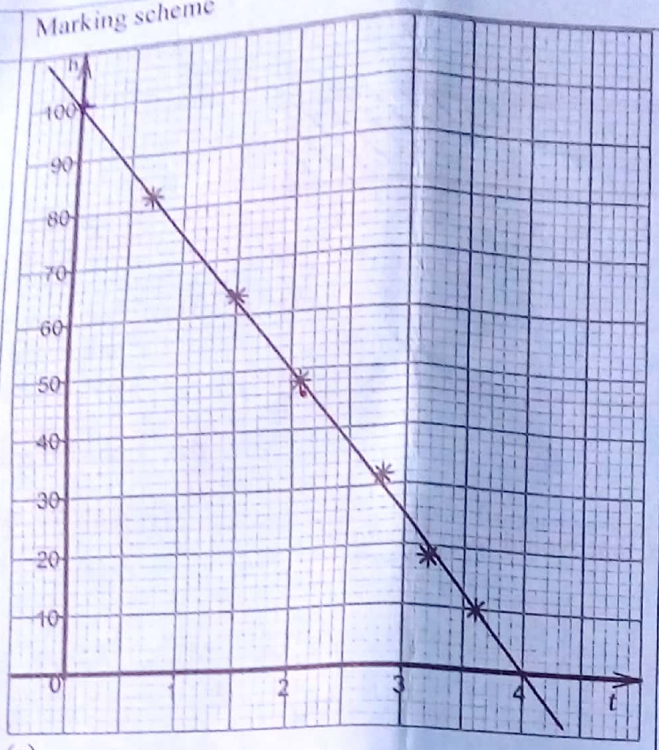
No.	Marking scheme	Marks	Comments
9.	 C	B1 — Line 3.5cm from AB and parallel to AB B1 — Semi circle, centre M, radius AM and broken arc B1 ✓ Shaded area 3	B1 B0 If continous arc has to be drawn B1✓ double tick if 2nd B is lost due to continuous c'
10.	NM = Single matrix that would map P onto P′ $= \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ $= \begin{pmatrix} 3 & 2 \\ 3 & 7 \end{pmatrix}$ $(NM)^{-1}$ = Single matrix that would map P′ onto P det = 21 − 6 = 15 $(NM)^{-1} = \frac{1}{15}\begin{pmatrix} 7 & -2 \\ -3 & 3 \end{pmatrix}$	M1 A1 B1 3	$\begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix}\begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ m1 $\begin{pmatrix} \frac{7}{15} & \frac{-2}{15} \\ -\frac{1}{5} & \frac{1}{5} \end{pmatrix}$ A1 B1 Allow $\begin{pmatrix} \frac{7}{15} & \frac{-2}{15} \\ \frac{-3}{15} & \frac{3}{15} \end{pmatrix}$

$\frac{1}{3}\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}\frac{1}{5}\begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix}$

$\frac{1}{15}\begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$ m1

$= \frac{1}{15}\begin{pmatrix} 7 & -2 \\ -3 & 3 \end{pmatrix}$

$\begin{pmatrix} \frac{7}{15} & \frac{-2}{15} \\ \frac{-3}{15} & \frac{3}{15} \end{pmatrix}$ A1 B1

No.	Marking scheme	Marks	Comments
11.			At 1.5, h = 64. and at 2.1, h = 46. (3.9 & 4)
	(a)	B1 P1	√ plotting of all 7 points coordinates
	(b)	B1 L1	√ line of best fit
	Rate of change of h with t $= \frac{100-50}{0-2}$ $= -25 cm/hr$.	B1	√ gradient
		3	
12.	$\Sigma d = 0$		
	$-4 + 5 + -3 + -2 + d + 1 = 0$		
	$d - 3 = 0$		
	$d = 3$	B1	
	Variance $= \frac{(-4)^2 + 5^2 + (-3)^2 + (-2)^2 + 3^2 + 1^2}{6}$	M1	
	$= \frac{64}{6}$		
	$= 10\frac{2}{3}$	A1	C.A.O
		3	

No.	Marking scheme	Marks	Comments
13.	Amount borrowed $= 27\,500 - 17\,250 = 10\,250$ Amount paid back $= 6 \times 2100 = 12\,600$ $10250\left(1 + \frac{r}{100}\right)^6 = 12600$	M1	
	$1 + \frac{r}{100} = \sqrt[6]{1.229}$	M1	for the 6th root.
	$= 1.035$		
	$r = 3.5\%$ p.m	A1	
		3	
14.	$\sin^2\theta - \cos^2\theta = -\frac{1}{2}$ $\sin^2\theta - (1 - \sin^2\theta) = -\frac{1}{2}$	M1	for substitution of $\cos^2\theta$ or equivalent
	$2\sin^2\theta = \frac{1}{2}$ $\sin^2\theta = \frac{1}{4}$ $\sin\theta = \pm\frac{1}{2}$	A1	Allow A1 for $\sin\theta = \frac{1}{2}$
	$\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$	B2	Allow B1 for 2 ~~or 3~~ for 2 angles.
		4	
15.	$\mathbf{PQ} = \begin{pmatrix}3\\3\\1\end{pmatrix} - \begin{pmatrix}1\\-1\\3\end{pmatrix} = \begin{pmatrix}2\\4\\-2\end{pmatrix}$ $\mathbf{PR} = \begin{pmatrix}6\\9\\-2\end{pmatrix} - \begin{pmatrix}1\\-1\\3\end{pmatrix} = \begin{pmatrix}5\\10\\-5\end{pmatrix}$		
	$\begin{pmatrix}5\\10\\-5\end{pmatrix} = k\begin{pmatrix}2\\4\\-2\end{pmatrix}$	M1	or equivalent
	$k = \frac{2}{5}$	A1	
	$\mathbf{PQ} = \frac{2}{5}\mathbf{PR}$. Thus $\mathbf{PQ} \parallel \mathbf{PR}$ P is a common point ~~P, Q and R are collinear.~~	B1	for Conclusion
		3	

No.	Marking scheme	Marks	Comments
16.	$S = \int_0^4 (t^2 - 4t + 6)\,dt$		
	$= \left[\frac{t^3}{3} - 2t^2 + 6t\right]_0^4$	M1	✓ integral with limits
	$= \left(\frac{64}{3} - 2\times16 + 6\times4\right) - 0$	M1	for substitution
	$= 13\frac{1}{3}$ M	A1	
		3	

Integrated no limits M1
my
A0.

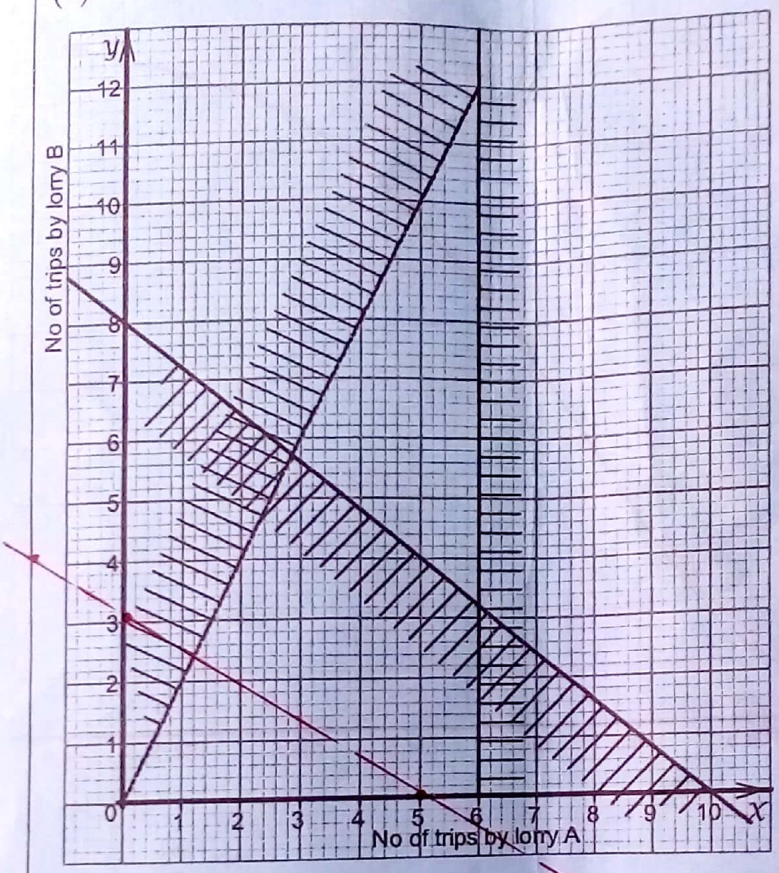
SECTION II (50 MARKS)

No.	Marking scheme	Marks	Comments
17.	(a) Tractor Q alone takes $(5-1\frac{2}{3}) = 3\frac{1}{3}$ h ✓	B1	for $3\frac{1}{3}$ hrs.
	Fraction of work done by tractor P and Q in 1 hour $= \frac{1}{5} + \frac{1}{3\frac{1}{3}} = \frac{1}{5} + \frac{3}{10}$ ✓	M1	
	$= \frac{1}{2}$	A1	
	Together P and Q take 2 hours		
	(b) Fraction of work done by P and Q in 40 minutes $= \frac{2}{3} \times \frac{1}{2}$ $= \frac{1}{3}$	B1	
	Balance $= 1 - \frac{1}{3} = \frac{2}{3}$		
	Tractor Q alone to do $\frac{2}{3}$ of work $= \frac{2}{3} \div \frac{3}{10} = \frac{2}{3} \times \frac{10}{3}$	M1	
	$= \frac{20}{9} = 2\frac{2}{9}$ hours		
	Total time $= 2\frac{2}{9} + \frac{2}{3}$	M1	
	$= 2\frac{8}{9}$ hours 2 hrs 53 min 20 sec.	A1	Allow 2hr 53
	(c) In 1 h both P and Q do $\frac{1}{2}$ of the work Fraction of work done by P and Q in 1h 12 min $= \frac{6}{5} \times \frac{1}{2} = \frac{3}{5}$	B1	for 2/5
	Balance $= 1 - \frac{3}{5} = \frac{2}{5}$		
	Payment for tractor R $= \frac{2}{5} \times 20\,000$	M1	
	= Ksh 8 000	A1	
		10	

No.	Marking scheme	Marks	Comments
18.	(a)(i) Moraa's monthly taxable income $= 40\,000 + 11\,090 + 7\,000$ $=$ ksh 58 090	~~B1~~ B1	
	(ii) Tax in 1st slab $= \frac{10}{100} \times 11180 = 1118$ Tax in 2nd slab $= \frac{15}{100} \times 10534 = 1580.1$	M1	
	Tax in 3rd slab $= \frac{20}{100} \times 10534 = 2106.8$ Tax in 4th slab $= \frac{25}{100} \times 10534 = 2633.5$	M1	
	Tax in 5th slab $= \frac{30}{100} \times 15308 = 4592.4$	M1	
	Total income tax $= 1118 + 1580.1 + 2106.8 + 2633.5 + 4592.4$ $= 12\,030.8$	A1	
	(b) Relief $= 12\,030.80 - 10\,750.8$ $=$ Ksh 1 280	B1	
	(c)(i) Tax in proposed 1st band $= 11180 \times 1.5 \times \frac{10}{100}$ $=$ ksh 1677 ✓	B1	$1118 \times \frac{150}{100}$
	(ii) Amount in last band $= 58090 - (16770 + 10534 \times 3)$ ✓ $= 9718$	M1	$\frac{150}{100} \times 11180 = 16770$ $16770 - 11180 = 5590$ $15308 - 5590 = 9718$
	$4592.4 - 1677$ Tax $= \frac{30}{100} \times 9718$ ✓ $= 2915.40$ ✓	M1 A1	Mark.
		10	

No.	Marking scheme	Marks	Comments
19.	(a)(i) Price of a pen $= \frac{180}{2x-1}$	B1	
	(ii) Price of a pencil $= \frac{200}{3x+1}$	B1	
	(b) $\frac{180}{2x-1} - \frac{200}{3x+1} = 4$	M1	or equivalent
	~~$180(3x+1) - 200(2x-1)(3x+1)$~~ $180(3x+1) - 200(2x-1) = 4(3x+1)(2x-1)$		
	$(2x-1)(3x+1) = 45(3x+1) - 50(2x-1)$		
	~~$24x^2 - 144$~~ $6x^2 - x - 1 = 35x + 95$	~~M1~~ M1	formation of Quadratic eq
	$24x^2 - 144x - 384 = 0$		
	$6x^2 - 36x - 96 = 0$		
	$x^2 - 6x - 16 = 0$		
	$(x+2)(x-8) = 0$	M1	✓ Complete factors ✓ attempt to solve
	$x = -2$ or $x = 8$		
	$x = 8$	A1	
	(c) New price of a pen $= \frac{125}{100} \times \left(\frac{180}{16-1}\right)$		$15m - 8n = 0$
	$=$ Ksh 15	B1	$m + n = 46$
	Price of pencil $= \frac{200}{25} =$ Ksh 8	B1	
	Let number of pens be p		
	$\therefore 15p = 8(46 - p)$	M1	← 1
	$15p + 8p = 8 \times 46$		
	$23p = 8 \times 46$		
	$p = \frac{8 \times 46}{23} = 16$	A1	
		10	

No.	Marking scheme	Marks	Comments
20.	(a)(i) Longitude difference between A and B		
	$= 15° + 75° = 90°$	B1	longitude diff
	$\frac{90}{360} \times 2 \times \frac{22}{7} \times 6370 \cos x = 5005$	M1	
	$\cos x = \frac{5005 \times 7 \times 360}{90 \times 2 \times 22 \times 6370} = 0.5000$		
	$x = 60°$		
	B(60°N, 75°W)	A1	
	(ii) Distance between B and C $= 910 \times 3\frac{2}{3} = 3336\frac{2}{3}$	M1	
	$\frac{\theta}{360} \times 2 \times \frac{22}{7} \times 6370 = 3336\frac{2}{3}$	M1	
	$\theta = \frac{3336\frac{2}{3} \times 360 \times 7}{2 \times 22 \times 6370} = 30°$		
	C(30°N, 75°W)	A1	60°-30°
	(b) Time for entire journey + stop over		
	$= \frac{5005}{910}$ + 1h 30 min + 3h 40 min	M1	- Total time take
	= 10 h 40 min		
	Time difference due to longitude difference		
	$= \frac{90 \times 4}{60} = 6\,h$	M1	
	Local time at C when aircraft reached		
	0720 − 0600		
	0120 + 1040	M1	
	1200 h	A1	
		10	

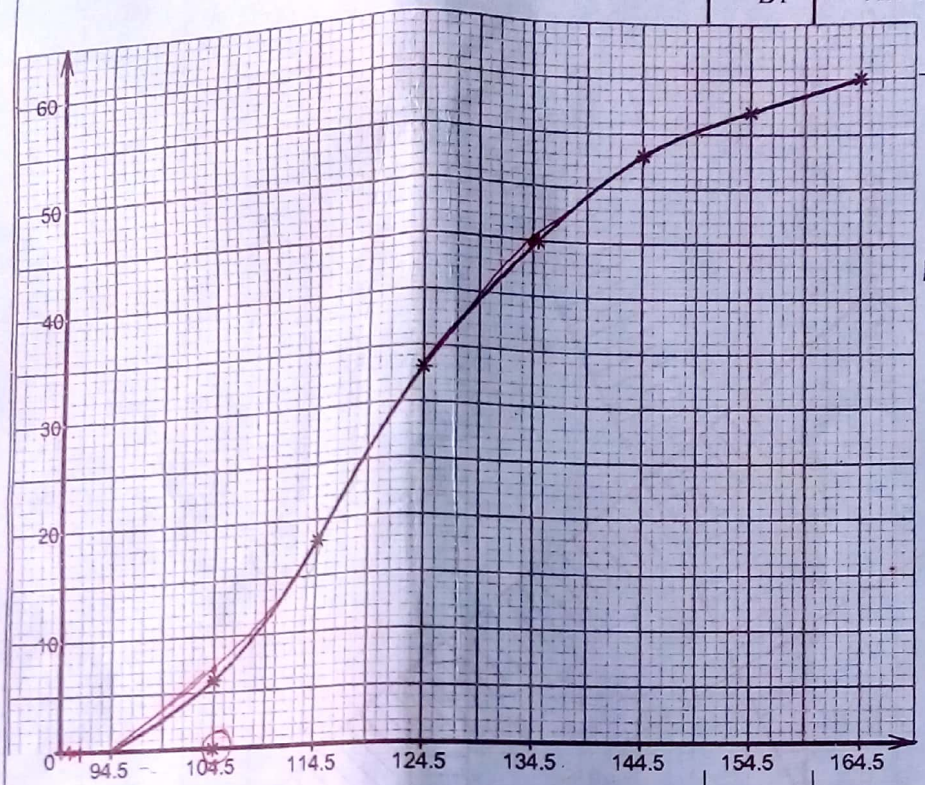
No.	Marking scheme	Marks	Comments
21.	(a) $y < 2x$	(i) B1	
	$3\times8\times x + 2\times15\times y \geq 240$ $24x + 30y \geq 240$ $4x + 5y \geq 40$	(ii) B1	
	$x \leq 6$	(iii) B1	
	(b) 	S1	
		B1	$y < 2x$ drawn and correct had.
		B1	$4x + 5y \geq 40$ drawn and ✓ shaded.
		B1	$x \leq 6$ drawn and ✓ shaded.
	(c) Search line		
	$3\times5000\times x + 2\times12500\times y = C$ $15000x + 25000y = C$ $15000x + 25000y = 75000$ $3x + 5y = 15$	B1	Search line drawn Any at least 3 value a mng ltd $4.
	For minimum cost, $x = 5, y = 4$	B1	
	Minimum cost $= 15000\times5 + 25000\times4$ $=$ Ksh 175 000	B1	
		10	

Alternatively (Inspection)
Must use three points with one of them being (5, 4)
$x = 5$ and $y = 4$.

No.	Marking scheme	Marks	Comments

22. (a)

Upper class boundary	104.5	114.5	124.5	134.5	144.5	154.5	164.5
Cumulative frequency	7	18	33	45	53	57	60



B1 | All √ ef

So P1 Co

104.5

S1
P1
C1

(b) Q_1 (Blood pressure of 15th patient) = 112 ± 0.5

Q3 (Blood pressure of 45th patient) = 134.5 ± 0.5 } B1 | Either Q_1 or Q_3 √

134.5 is exact

Range = 134.5 − 112 ✓ | M1 | If C is lost M0

= 22.5 ✓ | A1

(c) 56th patient is the 1st patient to exceed pressure of 150

No. of patients exeeding pressure of 150 = 5 | B1

Percentage = $\frac{5}{60} \times 100$ ✓ | M1

$= 8\frac{1}{3}\%$ ✓ $(8.333)\%$ | A1 | No decimals

$150 = 144.5 + \frac{(n-53)10}{4}$

$n = 55$.

$60 - 55 = 5$.

$\frac{5}{60} \times 100 = 8\frac{1}{3}$

10

No.	Marking scheme	Marks	Comments
23.	(a)(i) $\angle EAD = 40°$ ($\angle$ in alt. segment)	B1	
	$\angle ADE = 180 - (40 + 45) = 95$ (sum of angle in Δ)		
	$\angle BDC = 40^\circ$ (alternate angle)		
	$\angle ADB = 180 - (95 + 40)$	M1	
	$= 45^\circ$	A1	
	(ii) $\angle BAD = 180° - (45° + 40°) = 95°$		
	$\angle BCD = 180° - 95° = 85°$	B1	for 80° or 90°
	$\angle BOC = 2x40°$		
	$= 80°$		
	$\angle OCB = \left(180° - 80° \times \frac{1}{2}\right) = 50°$	~~B1~~ M1	$\left(\frac{180-110}{2}\right)$
	$\angle OCD = 85° - 50° = 35°$	~~B1~~ A1	
	(b)(i) $EA = \sqrt{3.5(3.5+4.9)} = \sqrt{3.5 \times 8.4}$	M1	
	$= 5.4$ cm	A1	
	(ii) $2r = \frac{4.9}{\sin 55°}$	M1	
	$r = 2.991$		
	$r \approx 3.0$ cm	A1	
			Follow thro'
		10	

No.	Marking scheme	Marks	Comments
24.	(a)(i) Total No. of students $= 60 + 56 + 44 + 40 = 200$	B1	Can be implied in the second
	$P(\text{Student in F4}) = \frac{40}{200} = \frac{1}{5}$	B1	
	(ii) P (Student wears glasses)		
	$= \frac{\frac{10}{100} \times 60 + \frac{12.5}{100} \times 56 + \frac{25}{100} \times 44 + \frac{17.5}{100} \times 40}{200}$	M1	
	$= \frac{6 + 7 + 11 + 7}{200}$		
	$= \frac{31}{200}$	A1	0.155
	(b)(i) P(Either F_1F_4 or F_4F_1)		
	$= \frac{60}{200} \times \frac{40}{199} + \frac{40}{200} \times \frac{60}{199}$	M1M1	Any one must be ✓.
	$= \frac{12}{199} + \frac{12}{199}$		
	$\frac{4800}{39800} = \frac{24}{199}$	A1	$\frac{4800}{39800}$
	(ii) P(Either F_1GF_4G or F_4GF_1G)		
	$= \frac{60}{200} \times \frac{10}{100} \times \frac{40}{199} \times \frac{17.5}{100} + \frac{40}{200} \times \frac{17.5}{100} \times \frac{60}{199} \times \frac{10}{100}$	M1M1	Any of these ✓ probabilities
	$= \frac{21}{19900} + \frac{21}{19900}$		
	$\frac{84}{39800} = \frac{42}{19900} = \frac{21}{9950}$	A1	$\frac{42}{39800}$
		10	

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