

THE KENYA NATIONAL EXAMINATIONS COUNCIL
Kenya Certificate of Secondary Education

121/2

Paper 2

MATHEMATICS
ALT A
Mar. 2022 – 2½ hours



Name Index Number

Candidate's Signature Date

M. J. Chumo

Instructions to candidates

- 280
- (a) Write your name and index number in the spaces provided above.
 - (b) Sign and write the date of examination in the spaces provided above.
 - (c) This paper consists of **two** sections: **Section I** and **Section II**.
 - (d) Answer **all** the questions in **Section I** and only **five** questions from **Section II**.
 - (e) **Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.**
 - (f) Marks may be given for correct working even if the answer is wrong.
 - (g) **Non-programmable** silent electronic calculators and KNEC Mathematical tables may be used except where stated otherwise.
 - (h) **This paper consists of 19 printed pages.**
 - (i) **Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.**
 - (j) **Candidates should answer the questions in English**

For Examiner's Use Only
Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	22	23	24	Total

Grand Total



Turn over

SECTION I (50 marks)

Answer all the questions in this section in the spaces provided.

1. An empty tank of capacity 18480 litres is to be filled with water using a cylindrical pipe of diameter 0.028 m. The rate of flow of water from the pipe is 2 m/s. Find the time in hours it would take to fill up the tank. (Take $\pi = \frac{22}{7}$). (3 marks)

Amount of water delivered by pipe in 1 sec.

$$= CA \times \text{rate} = \frac{22}{7} \times 0.014^2 \times 2 \times 1000$$

$$= 1.232 \text{ litres}$$

$$\text{Time taken to fill tank} = \frac{18480}{1.232} \times \frac{1}{3600}$$

$$= 4\frac{1}{6} \text{ hours (or } 4.167 \text{ hours (4sf)).}$$

2. The first term of a Geometric Progression (G.P) is 2. The common ratio of the G.P is also 2. The product of the last two terms of the G.P is 512. Determine the number of terms in the G.P. (3 marks)

let No. of terms be n .

$$a = 2, r = 2, T_n = ar^{n-1}$$

$$\text{Last term} = 2(2^{n-1})$$

$$\text{2nd last term} = 2(2^{n-2})$$

$$\text{Product} = 2 \times 2(2^{n-1})(2^{n-2}) = 512$$

$$\Rightarrow 4(2^{(n-1)+(n-2)}) = 512$$

$$\therefore 2^{2n-3} = 128 = 2^7$$

$$\Rightarrow 2n-3 = 7$$

$$\text{hence } n = 5$$

3. The expression $ax^2 - 30x + 9$ is a perfect square, where a is a constant. Find the value of a . (2 marks)

Alt. 1

Using discriminant:

$$b^2 = 4ac$$

$$(-30)^2 = 4(a)(9)$$

$$\Rightarrow a = 25.$$

Alt. 2

$$\text{let } ax^2 - 30x + 9 = (kx - b)^2$$

$$ax^2 - 30x + 9 = k^2x^2 - 2kbx + b^2$$

Comparing,

$$9 = b^2 \Rightarrow b = \pm 3$$

$$\text{and } 30 = 2kb \Rightarrow k = \pm 5$$

$$\text{and } a = k^2 \Rightarrow a = 25$$

4. Make x the subject of the formula $y = \frac{bx}{\sqrt{cx^2 - a}}$. (3 marks)

Squaring both sides,

$$y^2 = \frac{b^2 x^2}{cx^2 - a}$$

$$\Rightarrow y^2(cx^2 - a) = b^2 x^2$$

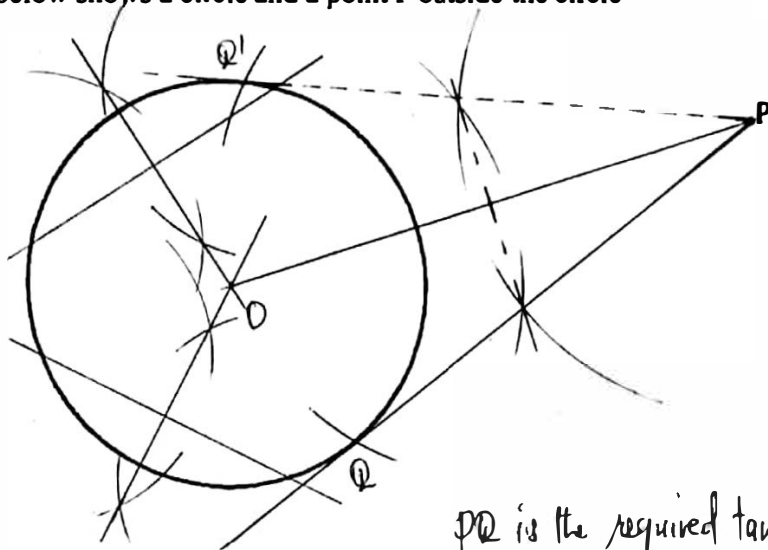
$$\Rightarrow y^2 cx^2 - b^2 x^2 = ay^2$$

$$\Rightarrow x^2 (cy^2 - b^2) = ay^2$$

$$x^2 = \frac{ay^2}{cy^2 - b^2}$$

$$x = \pm \sqrt{\frac{ay^2}{cy^2 - b^2}}$$

5. The figure below shows a circle and a point P outside the circle



PQ is the required tangent (PQ = 7.8 cm)

Using a ruler and pair of compasses, construct a tangent to the circle from P. (4 marks)

6. Four quantities P, Q, R and S are such that P varies directly as the square root of Q and inversely as the square of the difference of R and S. Quantity Q is increased by 44% while quantities R and S are each decreased by 10%.

Find the corresponding percentage change in P correct to 1 decimal place. (4 marks)

$$P \propto \frac{\sqrt{Q}}{(R-S)^2} \Rightarrow P = \frac{k\sqrt{Q}}{(R-S)^2}$$

$$\% \Delta \text{ in } P = \frac{\frac{40}{27} - 1}{1} \times 100$$

$$P' = \frac{k\sqrt{1.44Q}}{[0.9(R-S)]^2}$$

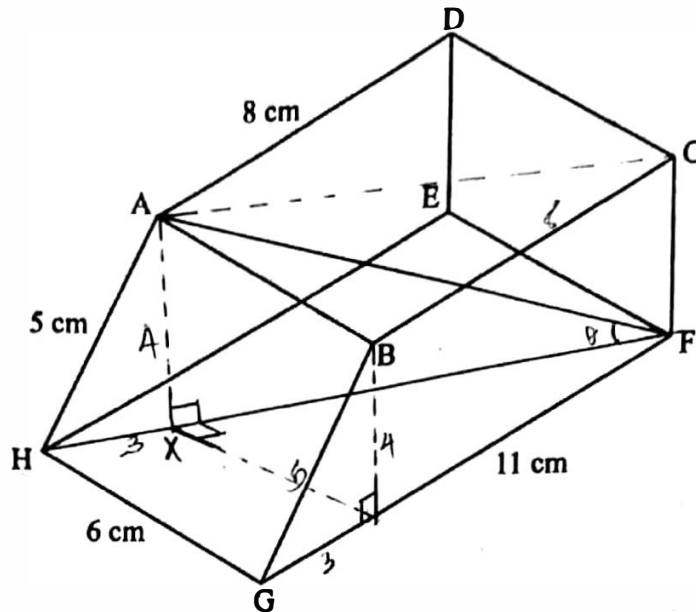
$$= 48.1\%$$

(Increase)

$$= \frac{1.2}{0.9^2} \frac{k\sqrt{Q}}{(R-S)^2}$$

$$\Rightarrow P' = \frac{40}{27} P$$

7. The figure below represents a prism ABCDEFGH of length 6 cm. The cross section BCFG of the prism is a trapezium in which $GF = 11$ cm, $BC = 8$ cm, $BG = 5$ cm and $\angle GFC = \angle BCF = 90^\circ$.



Calculate correct to 1 decimal place the angle between the line FA and the plane GFEH.

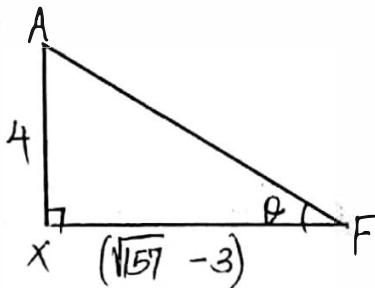
(3 marks)

$$FH = \sqrt{6^2 + 11^2} = \sqrt{157}$$

$$\tan \theta = \frac{4}{\sqrt{157} - 3}$$

$$\theta \approx 22.8^\circ$$

(or $\theta \approx 21.8^\circ$ if $FX = AC = 10$ cm is used)



8. The cash price of a gas cooker is Ksh 20 000. A customer bought the cooker on hire purchase terms by paying a deposit of Ksh 10 000 followed by 18 equal monthly instalments of Ksh 900 each. Annual interest, compounded quarterly, was charged on the balance for the period of 18 months. Determine, correct to 1 decimal place, the rate of interest per annum. (4 marks)

$$A = P \left(1 + \frac{r}{100}\right)^n$$

$$(18 \times 900) = (20000 - 10000) \left(1 + \frac{r}{400}\right)^{\frac{18 \times 4}{12}}$$

$$\sqrt[6]{\frac{16200}{10000}} = \left(1 + \frac{r}{400}\right)$$

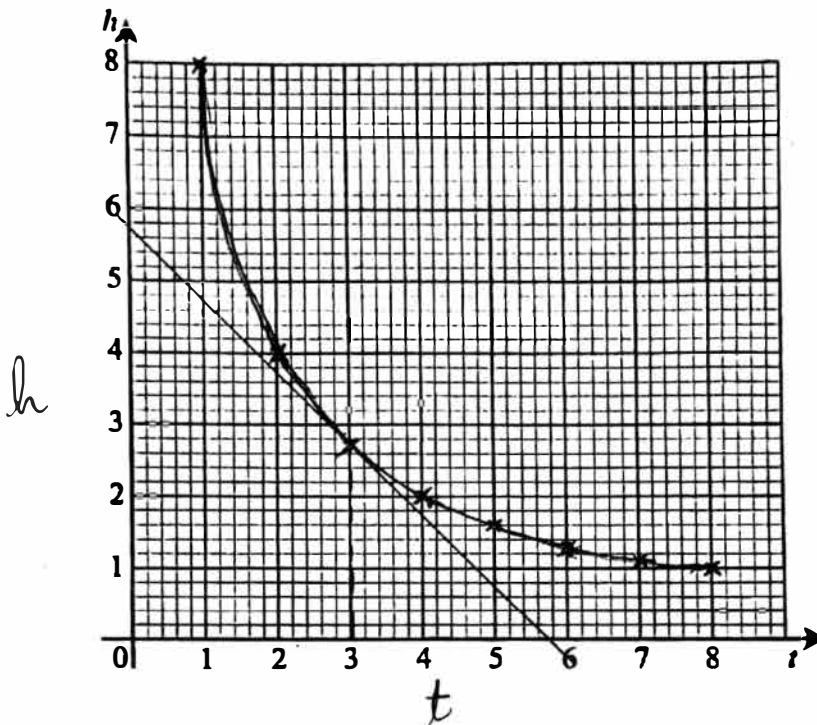
$$r = \left(\sqrt[6]{1.62} - 1\right) \times 400$$

$$= \underline{\underline{33.5\%}}$$

9. The table below shows the values of t and the corresponding values of h for a given relation.

t	1	2	3	4	5	6	7	8
h	8	4	2.7	2	1.6	1.3	1.1	1

- (a) On the grid provided, draw a graph to represent the information on the table given. (2 marks)



- (b) Use the graph to determine, correct to 1 decimal place, the rate of change of h at $t = 3$. (2 marks)

$$\begin{aligned}
 \text{Rate of change} &= \text{gradient at } t = 3 \\
 &= \frac{\Delta y}{\Delta x} \\
 &= \frac{0 - 5.7}{5.7 - 0} \\
 &= -1.0
 \end{aligned}$$

10. The equation of a trigonometric wave is $y = 4 \sin(ax - 70)^\circ$. The wave has a period of 180° .

(a) Determine the value of a. (1 mark)

$$\frac{360}{a} = 180^\circ \Rightarrow a = 2$$

(b) Deduce the phase angle of the wave. (1 mark)

$$|-70^\circ| \Rightarrow 70^\circ$$

11. A point Q is 2000 nm to the West of a point P(40°N , 155°W). Find the longitude of Q to the nearest degree. (3 marks)



$$2000 = 60 \times \theta \cos 40^\circ$$

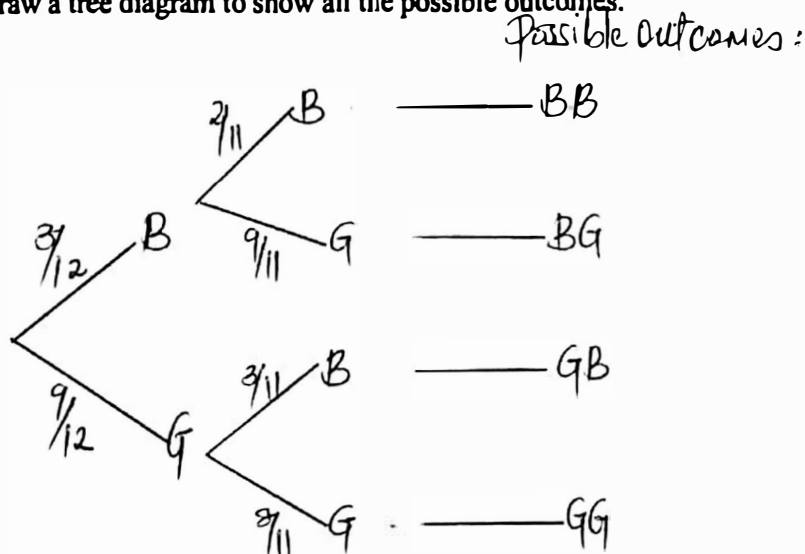
$$\theta = 43.51^\circ$$

$$\text{Longitude} = 180 - (43.51 - 25)$$

$$\approx 161^\circ \text{E}$$

12. A box contains 3 brown balls and 9 green balls. The balls are identical except for the colours. Two balls are picked at random without replacement.

(a) Draw a tree diagram to show all the possible outcomes. (1 mark)



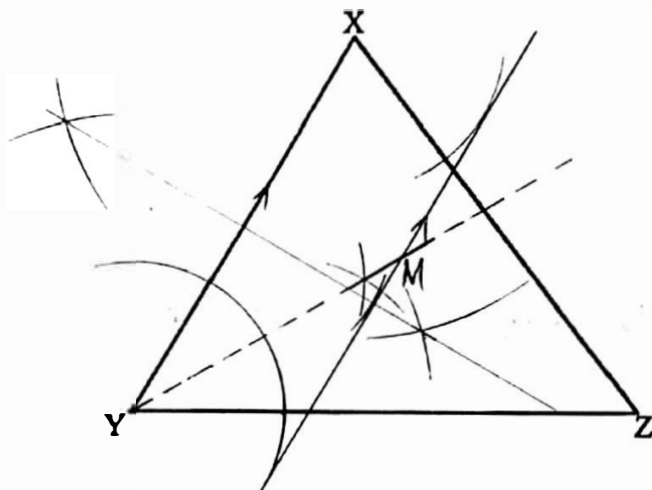
- (b) Determine the probability that the balls picked are of different colours. (2 marks)

$$P(BG) + P(GB)$$

$$\left(\frac{3}{12} \times \frac{9}{11}\right) + \left(\frac{9}{12} \times \frac{3}{11}\right)$$

$$\frac{54}{132} \quad \text{or} \quad \frac{9}{22}$$

13. The figure below shows triangle XYZ.



Using a ruler and a pair of compasses, locate a point M on the triangle such that M is 2 cm from line YX and is equidistant from lines YX and YZ. Measure length YM. (3 marks)

$$YM = 4.1 \text{ cm}$$

14. The position vectors of points P, Q and R are $\underline{OP} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\underline{OQ} = 12\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}$ and $\underline{OR} = 8\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$. Show that P, Q and R are collinear points. (3 marks)

$$\begin{array}{c} \text{P} \quad \quad \text{Q} \quad \quad \text{R} \\ \hline \end{array}$$

$$\underline{PQ} = \underline{OQ} - \underline{OP} = (12\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}) - (6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$$

$$= 6\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$$

$$\underline{PR} = \underline{OR} - \underline{OP} = (8\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) - (6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$$

$$= 2\mathbf{i} - \mathbf{j} + \mathbf{k}$$

$$\underline{PQ} = 3\underline{PR} \Rightarrow \underline{PQ} \parallel \underline{PR} \text{ and point P is common.}$$

hence P, Q and R are collinear points.

Alt. 2

$$\underline{PQ} = \begin{pmatrix} 12 \\ -5 \\ 6 \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix}$$

$$\underline{PR} = \begin{pmatrix} 8 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\underline{PQ} = 3\underline{PR}$$

15. In a transformation an object of area $x \text{ cm}^2$ is mapped on to an image whose area is $13x \text{ cm}^2$.

Given that the matrix of the transformation is $\begin{pmatrix} x & 7 \\ x-1 & 3x \end{pmatrix}$. Find the possible values of x .
(3 marks)

$$\text{ASF} = \frac{\det. \text{ of transf. Matrix}}{\text{A. of Object}} = \frac{\text{A. of Image}}{\text{A. of Object}}$$

$$\Rightarrow x(3x) - 7(x-1) = \frac{13x}{x} = 13$$

$$\Rightarrow 3x^2 - 7x + 7 - 13 = 0$$

$$\Rightarrow 3x^2 - 7x - 6 = 0$$

$$\Rightarrow 3x^2 - 9x + 2x - 6 = 0$$

$$\Rightarrow (x-3)(3x+2) = 0$$

$$\Rightarrow x-3 = 0 \text{ or } 3x+2 = 0 \Rightarrow x = 3 \text{ or } x = -\frac{2}{3}$$

16. Find the area enclosed by the curve $y = x^2 + 2x$ the straight lines $x = 1$, $x = 3$ and the x -axis.
(3 marks)

$$\frac{dy}{dx} = 2x + 2 = 0$$

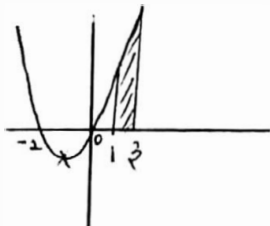
$$x = -1$$

$$y = -1$$

T.p at $(-1, -1)$

$$x(x+2) = 0$$

$$x = 0 \text{ or } x = -2$$



$$A = \int y \, dx$$

$$= \int_1^3 (x^2 + 2x) \, dx$$

$$= \left[\frac{x^3}{3} + x^2 + c \right]_1^3$$

$$= \left(\frac{27}{3} + 9 + c \right) - \left(\frac{1}{3} + 1 + c \right)$$

$$= 16\frac{2}{3} \text{ Sq. Units.}$$

SECTION II (50 marks)

Answer only five questions in this section in the spaces provided.

17. Pump P can fill an empty water tank in $7\frac{1}{2}$ hours while pump Q can fill the same tank in $11\frac{1}{4}$ hours. On a certain day, when the tank was empty, both pumps were opened for $2\frac{1}{2}$ hours.

- (a) Determine the fraction of the tank that was still empty at the end of the $2\frac{1}{2}$ hours. (4 marks)

	Pump P	Pump Q
Time to fill (h) :	$15\frac{1}{2}$	$45\frac{1}{4}$
Rate of filling (h^{-1}) :	$\frac{2}{15}$	$\frac{4}{45}$

Together for $\frac{5}{2}$ hrs fill $\frac{5}{2} \left(\frac{2}{15} + \frac{4}{45} \right) = \frac{5}{9}$ of tank

Fraction of tank still empty = $1 - \frac{5}{9} = \frac{4}{9}$

- (b) Pump P was later opened alone to completely fill the tank. Determine the time it took pump P to fill the remaining fraction of the tank. (2 marks)

$\frac{2}{15}$ of tank is filled in 1 hr ;

$\frac{4}{9}$ of tank will be filled in $\left(\frac{4}{9} \times 15\frac{1}{2} \times 1 \right) = 3\frac{1}{3}$ hrs.

- (c) The two pumps P and Q are operated by different proprietors. Water from the full tank was sold for Ksh 15 750. The money was shared between the two proprietors in the ratio of the quantity of water supplied by each.

Determine the amount of money received by the proprietor of pump P. (4 marks)

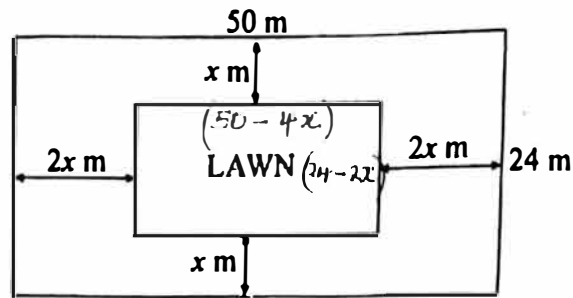
proportion of P : Q = $\frac{2}{15} : \frac{4}{45}$

= 6 : 4

Proprietor P received $\frac{6}{10} \times 15\,750$

= Ksh 9 450

18. A rectangular plot measures 50 m by 24 m. A lawn, rectangular in shape, is situated inside the plot with a path surrounding it as shown in the figure below.



The width of the path in x m between the lengths of the lawn and those of the plot and $2x$ m between the widths of the lawn and those of the plot.

- (a) Form and simplify an expression in x for the area of the:

- (i) lawn;

(2 marks)

$$(50 - 4x)(24 - 2x)$$

$$8x^2 - 196x + 1200$$

- (ii) path.

(1 mark)

$$(50 \times 24) - (8x^2 - 196x + 1200)$$

$$-8x^2 + 196x$$

(b) The area of the path is $1\frac{1}{2}$ times the area of the lawn.

(i) Form an equation in x and hence solve for x .

(4 marks)

$$-8x^2 + 196x = 1.5(8x^2 - 196x + 1200)$$

$$-20x^2 + 490x - 1800 = 0$$

$$\text{or } 2x^2 - 49x + 180 = 0$$

$$(2x-9)(x-20) = 0$$

$$2x-9 = 0 \text{ or } x-20 = 0$$

$$\Rightarrow x = 4\frac{1}{2} \text{ or } x = 20$$

(ii) Determine the perimeter of the lawn.

(3 marks)

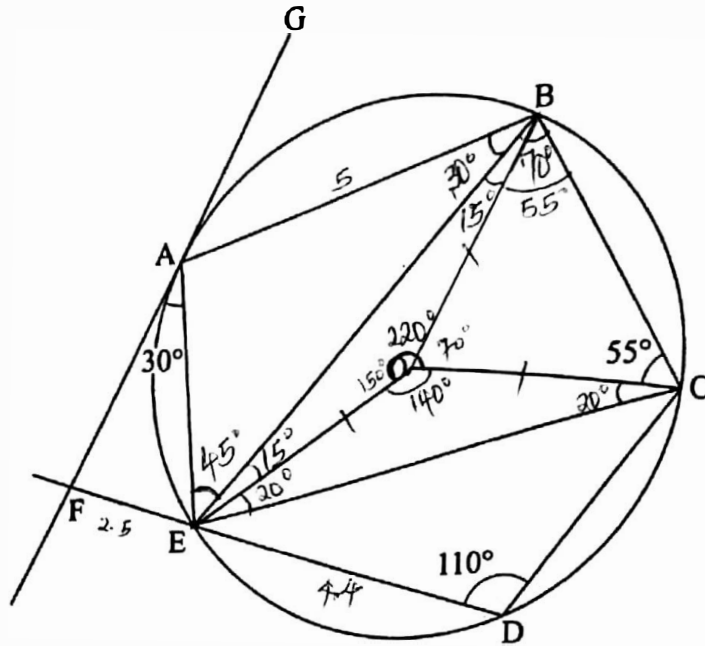
appropriate value of $x = 4\frac{1}{2}$

$$\text{Length of lawn} = 50 - 4 \times 4\frac{1}{2} = 32 \text{ M}$$

$$\text{Width of lawn} = 24 - 2 \times 4\frac{1}{2} = 15 \text{ M}$$

$$\text{perimeter} = 2(32 + 15) = 94 \text{ M}$$

19. In the figure below, points A, B, C, D and E lie on the circumference of a circle centre O. Line FAG is a tangent to the circle at A. Chord DE of the circle is produced to intersect with the tangent at F. Angle FAE = 30° , $\angle EDC = 110^\circ$ and $\angle OCB = 55^\circ$.



- (a) Determine the size of:

(i) $\angle AEC = 180^\circ - (70 + 30)^\circ$
 $= 80^\circ$ (3 marks)

(ii) $\angle AEB = 80^\circ - (20 + 15)^\circ$
 $= 45^\circ$ (3 marks)

- (b) Given that $AB = 5$ cm, $ED = 4.4$ cm and $FE = 2.5$ cm. Calculate correct to 1 decimal place:

- (i) the radius of the circle. (2 marks)

$$\frac{5}{\sin 45} = 2R$$

$$R = 3.5 \text{ cm}$$

- (ii) the length of line AF. (2 marks)

$$DF \cdot FE = AF^2$$

$$2.5(4.4 + 2.5) = AF^2$$

$$\Rightarrow AF = 4.2 \text{ cm}$$

20. The table below shows income tax rates in a certain year.

Monthly taxable income in Kenya shillings	Tax rates
0 – 12 298	10%
12 299 – 23 885	15%
23 886 – 35 472	20%
35 473 – 47 059	25%
47 060 and above	30%

In the year, the monthly earnings of Kanini were as follows:

Basic salary	Ksh 64 500
House allowance	Ksh 12 000

Kanini contributes 7.5 % of her basic salary to a pension scheme. This contribution is exempted from taxation. She is entitled to a personal tax relief of Ksh 1 408 per month.

Calculate:

- (a) Kanini's monthly taxable income. (2 marks)

$$= 64\,500 + 12\,000 - \frac{7.5}{100} \times 64\,500$$

$$= \text{Ksh } 71\,662.50$$

- (b) the tax payable by Kanini that month. (6 marks)

$$\begin{aligned} 1^{\text{st}} \text{ band} &: 10\% \times 12\,298 = \text{Ksh } 1\,229.80 \\ 2^{\text{nd}} \text{ band} &: 15\% \times 11\,587 = \text{Ksh } 1\,738.05 \\ 3^{\text{rd}} \text{ band} &: 20\% \times 11\,587 = \text{Ksh } 2\,317.40 \\ 4^{\text{th}} \text{ band} &: 25\% \times 11\,587 = \text{Ksh } 2\,896.75 \\ 5^{\text{th}} \text{ band} &: 30\% \times 24\,603.50 = \text{Ksh } 7\,381.05 \\ \hline \text{Gross tax} &= \text{Ksh } 15\,563.05 \end{aligned}$$

- (c) Kanini's net pay that month. (2 marks)

$$* (\text{Net tax} = \text{Ksh } (15\,563.05 - 1\,408) = \text{Ksh } 14\,155.05)$$

$$\text{Net tax} = \text{Ksh } 14\,155.05$$

$$\text{Pension} = \text{Ksh } 4\,837.50 \uparrow$$

$$\text{Total deductions} = \text{Ksh } 18\,992.55$$

$$\text{Net pay} = 76\,500 - 18\,992.55$$

$$= \text{Ksh } 57\,507.45$$

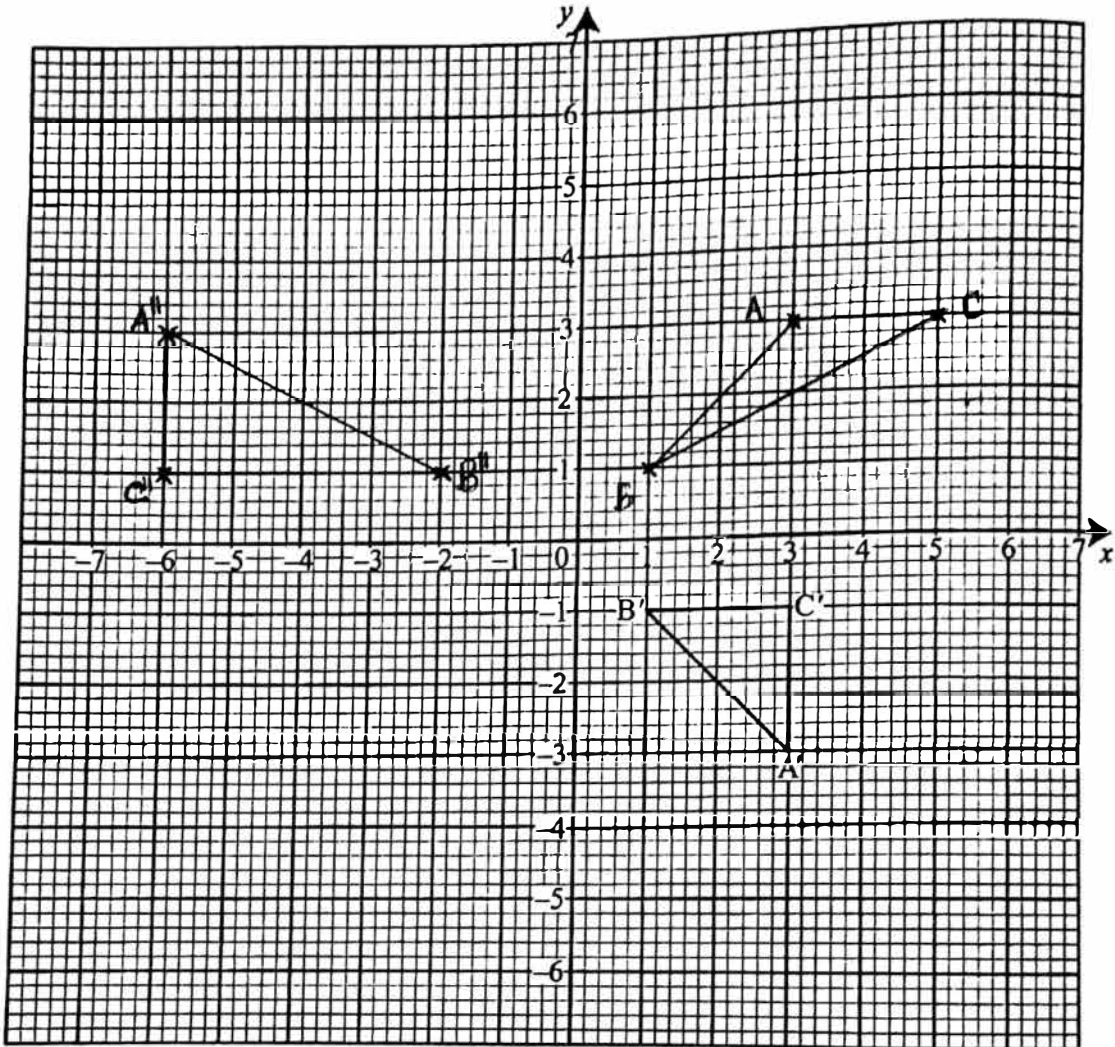
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21. The vertices of the triangle shown on the grid are $A'(3,-3)$, $B'(1,-1)$ and $C'(3,-1)$.

Triangle $A'B'C'$ is the image of triangle ABC under a transformation whose matrix is $\begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$.



- (a) Find the coordinates of triangle A , B and C .

(4 marks)

$$\begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} 3 & 1 & 3 \\ -3 & -1 & -1 \end{pmatrix}$$

$$\begin{aligned} 0a + b &= 3 \Rightarrow b = 3 & 0c + d &= 1 \Rightarrow d = 1 & 0e + f &= 3 \Rightarrow f = 3 \\ a - 2b &= -3 \Rightarrow a = 3 & c - 2d &= -1 \Rightarrow c = 1 & e - 2f &= -1 \Rightarrow e = 5 \end{aligned}$$

$$\Rightarrow A(3,3), B(1,1), C(5,3)$$

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Using

$$\text{Inv.} \Rightarrow \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} A' & B' & C' \\ 3 & 1 & 3 \\ -3 & -1 & -1 \end{pmatrix} = \begin{pmatrix} A & B & C \\ 3 & 1 & 5 \\ 3 & 1 & 3 \end{pmatrix}$$

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 $\Rightarrow A(3,3), B(1,1), C(5,3)$

- (b) Triangle $A''B''C''$ is the image of triangle $A'B'C'$ under a transformation matrix

$$\begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}$$

(2 marks)

Determine the coordinates of A'' , B'' and C'' .

$$\begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 3 & 1 & 3 \\ -3 & -1 & -1 \end{pmatrix} = \begin{pmatrix} -6 & -2 & -6 \\ 3 & 1 & 1 \end{pmatrix}$$

$$\Rightarrow A''(-6, 3), B''(-2, 1), C''(-6, 1)$$

- (c) On the same grid provided, draw triangles ABC and $A''B''C''$.

(2 marks)

- (d) Determine a single matrix that maps ABC onto $A''B''C''$.

(2 marks)

$$ABC \xrightarrow{\underline{NM}} A''B''C''$$

$$\xleftarrow{(\underline{NM})^{-1}}$$

$$\text{let } \underline{M} = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}, \underline{N} = \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\underline{NM} = \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ -1 & 2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 & -2 \\ -1 & 2 \end{pmatrix}$$

22. Workers in a factory commute from their homes to the factory. The table below shows the distances in kilometres, covered by the workers.

Distance (km)	1 - 5	6 - 10	11 - 15	16 - 20	21 - 25	26 - 30
Number of workers	3	6	1	7	4	2

The mean distance covered was 14.5 km.

- (a) Determine the value of t and hence the standard deviation of the distances correct to 2 decimal places. (6 marks)

Distance	f	x	fx	$d = x - 14.5$	fd^2
1 - 5	3	3	9	-11.5	376.75
6 - 10	6	8	48	-6.5	253.5
11 - 15	$t=8$	13	$13t$	-1.5	18
16 - 20	7	18	126	3.5	85.75
21 - 25	4	23	92	8.5	289
26 - 30	2	28	56	13.5	364.5
	<u>$22+t$</u>		<u>$331+13t$</u>		<u>1407.5</u>

(i) $\frac{331+13t}{22+t} = 14.5$
 $331+13t = 14.5(22+t)$
 $t = 8$

(ii) $s = \sqrt{\frac{\sum fd^2}{\sum f}}$
 $= \sqrt{\frac{1407.5}{30}}$
 $= 6.85 \text{ km}$

- (b) Calculate, correct to 2 decimal places, the interquartile range of the distances. (4 marks)

Distance	f	cf
1 - 5	3	3
6 - 10	6	9
11 - 15	8	17
16 - 20	7	24
21 - 25	4	28
26 - 30	<u>2</u>	<u>30</u>

$Q_1 = \frac{1}{4} \times 30 = 7.5^{\text{th}}$
 $= 5.5 + \left(\frac{17.5-3}{6}\right) \times 5 = 9.25 \text{ km}$

$Q_3 = \frac{3}{4} \times 30 = 22.5^{\text{th}}$
 $= 15.5 + \left(\frac{22.5-17}{7}\right) \times 5 = 19.43 \text{ km}$

$Q_3 - Q_1 = 19.43 - 9.25 = 10.18 \text{ km}$

23. (a) Complete the table below giving the values correct to 1 decimal place.

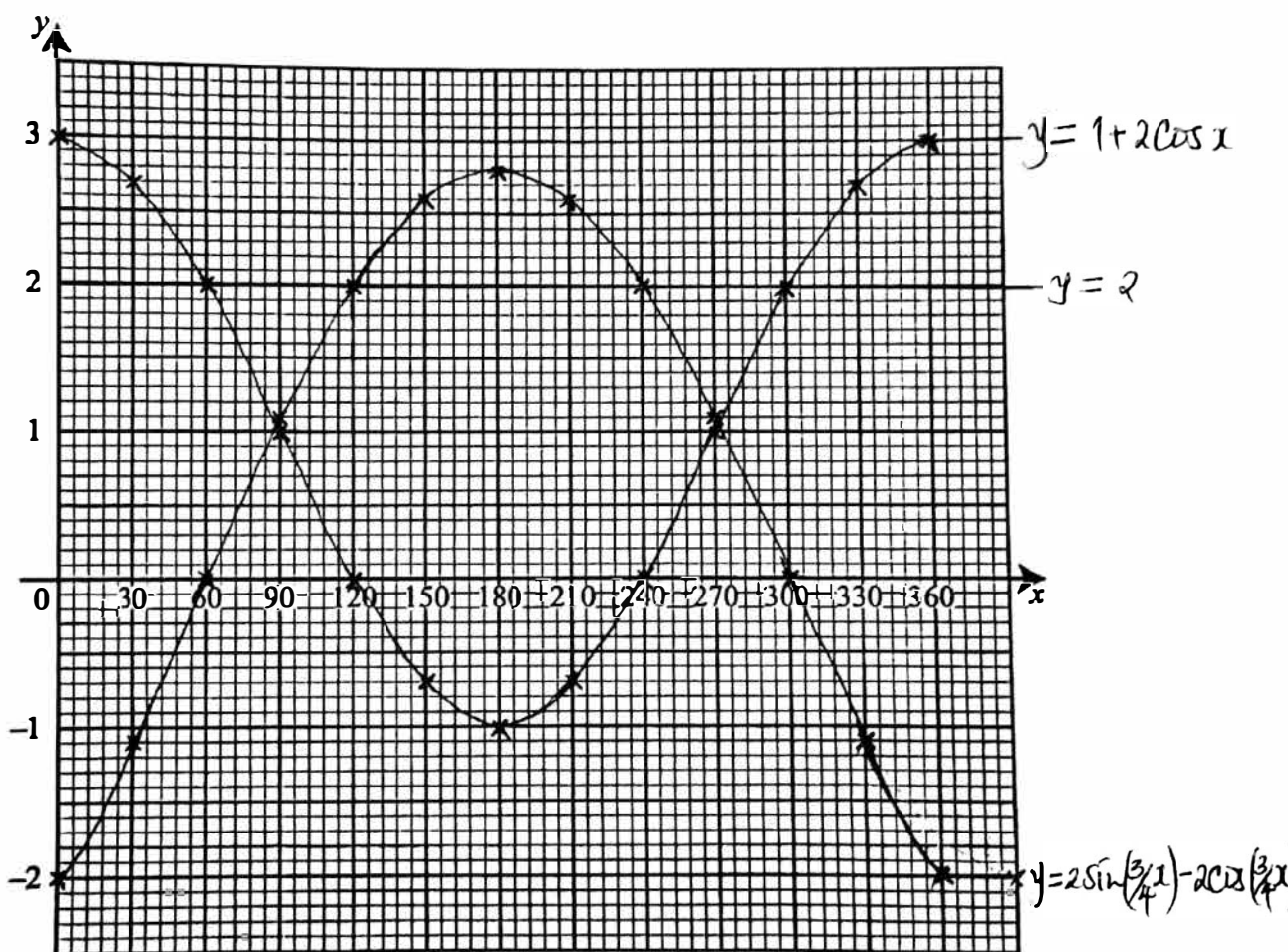
x°	0	30	60	90	120	150	180	210	240	270	300	330	360
$y = 2\sin\left(\frac{3}{4}x\right) - 2\cos\left(\frac{3}{4}x\right)$	-2	-1.1	0	1.1	2	2.6	2.8	2.6	2	1.1	0	-1.1	-2
$y = 1 + 2\cos x$	3	2.7	2	1	0	-0.7	-1	-0.7	0	1	2	2.7	3

(2 marks)

(b) On the grid provided and using the same axis, draw the graphs of

$y = 2\sin\left(\frac{3}{4}x\right) - 2\cos\left(\frac{3}{4}x\right)$ and $y = 1 + 2\cos x$ for $0^\circ \leq x \leq 360^\circ$.

(4 marks)



$$\sin\left(\frac{3x}{4}\right) - \cos\left(\frac{3x}{4}\right) = 1$$

(c) Using the graphs in part (b):

(i) find the values of x for which $\sin\left(\frac{3}{4}x\right) = 1 + \cos\left(\frac{3}{4}x\right)$. (2 marks)

$$y = 2 \left[\sin\left(\frac{3}{4}x\right) - \cos\left(\frac{3}{4}x\right) \right]$$

$$\frac{y}{2} = \left[\sin\left(\frac{3}{4}x\right) - \cos\left(\frac{3}{4}x\right) \right] = 1$$

$$y = 2$$

(ii) determine the range of x for which $2\sin\left(\frac{3}{4}x\right) - 2\cos\left(\frac{3}{4}x\right) > 1 + 2\cos x$. (2 marks)

$$89^\circ < x < 271^\circ$$

24. A particle was moving along a straight line. The acceleration of the particle after t seconds was given by $(4t - 13) \text{ ms}^{-2}$. The initial velocity of the particle was 18 ms^{-1} .

(a) Determine the value of t when the particle is momentarily at rest. (5 marks)

$$\begin{aligned}
 v &= \int a \, dt \\
 &= \int (4t - 13) \, dt \\
 &= 2t^2 - 13t + c
 \end{aligned}$$

at initial velocity, $t=0, v=18 \Rightarrow c=18$

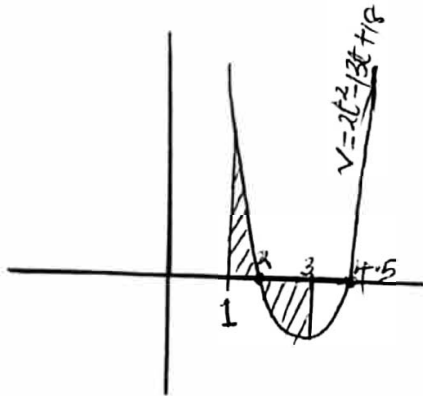
$$v = 2t^2 - 13t + 18$$

But $v=0 = 2t^2 - 13t + 18$

$$\Rightarrow (t-2)(2t-9) = 0$$

$$t-2=0 \text{ or } 2t-9=0 \quad \Rightarrow t=2 \text{ sec. or } t=4.5 \text{ sec.}$$

(b) Find the distance covered by the particle between the time $t=1$ second and $t=3$ seconds. (5 marks)



$$\frac{dv}{dt} = 4t - 13 = 0$$

$$t = \frac{13}{4}$$

$$v = -3\frac{1}{4}$$

$$\text{T.p. } (3\frac{1}{4}, -3\frac{1}{4})$$

$$\begin{aligned}
 s &= \int v \, dt \\
 &= \int_1^2 (2t^2 - 13t + 18) \, dt + \left| \int_2^3 (2t^2 - 13t + 18) \, dt \right| \\
 &= \left[\frac{2}{3}t^3 - \frac{13}{2}t^2 + 18t + c \right]_1^2 + \left| \left[\frac{2}{3}t^3 - \frac{13}{2}t^2 + 18t + c \right]_2^3 \right| \\
 &= 3\frac{1}{6} + |-1\frac{5}{6}| \\
 &= 3\frac{1}{6} + 1\frac{5}{6} \\
 &= 5 \text{ Metres.}
 \end{aligned}$$

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