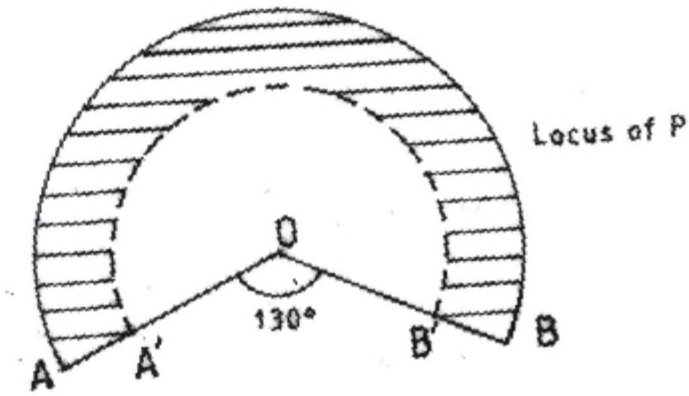
2013 Paper 2

4.3.2 Mathematics Alternative A Paper 2 (121/2)

1.	1^{st} term, a = 3; common difference, d = 6 $7500 = \frac{n}{2}\{2 \times 3 + (n-1) \times 6\}$ $3n^2 = 7500$ $n = \sqrt{2500} = 50$	B1 M1 A1	
		3	
2.	$y = (x + 2)(x - 1)$ $y = x^2 + x - 2$	M1 A1	
		2	
3.	$P = \frac{1}{2}mn^2 - \frac{qd^2}{n}$ $\frac{qd^2}{n} = \frac{1}{2}mn^2 - P$ $d^2 = \frac{\frac{1}{2}mn^3 - nP}{q}$ $d = \sqrt{\frac{\frac{1}{2}mn^3 - nP}{q}}$	 M1 M1 A1	
		3	
4.	$Log\left(\frac{x^2}{x-2}\right) = \log 3^2$ $\frac{x^2}{x-2} = 9$ $x^2 - 9x + 18 = 0$ $(x - 6)(x - 3) = 0$ $x = 6$ or $x = 3$	M1 M1 A1	
		3	

5.	(a)	B1	extending YX and YZ
		B1	bisecting +S VXZ and XZW
		B1	escribed circle drawn
	(b) radius = 3.1	B1	allow ± 0.1
		4	
6.	Completing square on L.H.S.		
	$x^2 + 4x + 4 + y^2 - 2y + 1 = 4 + 4 + 1$	B1	
	$(x + 2)^2 + (y - 1)^2 = 9$	B1	
	` centre of circle : (-2, 1)	B1	
	radius of circle: 3 units 4		
		3	
7.	(a) $(1 - x)^5 = 1 + 5(-x) + 10(-x)^2 + 10(-x)^3 + 5(-x)^4 + (-x)^5$		
	$= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$	B1	
	(b) $(0.98)^5 = (1 - 0.02)^5$ & $x = 0.02$		
	` $(0.98)^5 = 1 - 5(0.02) + 10(0.02)^2 - 10(0.02)^3$	M1	
	$= 1 - 0.1 + 0.004 - 0.00008$		
	$= 0.90392$	A1	
		3	

8.	$\underset{\sim}{h} = \frac{-1}{4+(-1)}\underset{\sim}{f} + \frac{4}{4+(-1)}\underset{\sim}{g}$ $= \frac{-1}{3}\underset{\sim}{f} + \frac{4}{3}\underset{\sim}{g}$	M1 A1 2	
9.	P(defective) : M $\rightarrow$ 0.6 $\times$ 0.05 = 0.03 N $\rightarrow$ 0.4 $\times$ 0.03 = 0.012 P(defective) 0.03 + 0.02 = 0.042	M1 M1 A1 3	For 0.6 $\times$ 0.05 or 0.4 $\times$ 0.03 M: 0.6 — 0.95 good; 0.05 defective N: 0.4 — 0.97 good; 0.03 defective
10.	(a) Fraction filled if A and R are open for 5h $5 \times \left(\frac{1}{3} - \frac{1}{6}\right) = \frac{5}{6}$ Fraction of tank still empty = $1 - \frac{5}{6} = \frac{1}{6}$ (b) Fraction filled if A, B and R are open for 1h $\frac{1}{3} + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}$ Time taken to fill the tank = $\frac{1}{6} \div \frac{2}{3} = \frac{1}{6} \times \frac{3}{2}$ $= \frac{1}{4}$h or 15 min	B1 B1 M1 A1 4	
11.	$\frac{\sqrt{48}}{\sqrt{5}+\sqrt{3}} = \frac{4\sqrt{3}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$ $= \frac{4\sqrt{3}(\sqrt{5}-\sqrt{3})}{5-3}$ $= 2\sqrt{3}(\sqrt{5}-\sqrt{3})$ $= 2\sqrt{15}-6$	M1 M1 A1 3	

12.	 $\angle AOB = 130^\circ$ arc AB - solid curve arc A′B′ - broken curve region shown	B1 B1 B1 B1 4	
13.	$9680 \times 0.1 = 968$ 9120×0.15; 9120×0.2; 4580×0.25 $= 1368$ $= 1824$ $= 1145$ Net tax $= (968 + 1368 + 1824 + 1145) - 1056$ $= 4249$	M1 M1 M1 A1 4	
14.	$6(1 - \sin^2 x) + 7 \sin x - 8 = 0$ $6 - 6 \sin^2 x + 7 \sin x - 8 = 0$ $6 \sin^2 x - 7 \sin x + 2 = 0$ $(3 \sin x - 2)(2 \sin x - 1) = 0$ $\sin x = \frac{2}{3}$ or $\sin x = \frac{1}{2}$ $x = 41.81^\circ$ or $x = 30^\circ$	M1 M1 M1 A1 4	

15.	Distance between towns K and S $= 2\pi \times 6370 \cos 2^\circ \times \frac{37.4 - 30}{360}$ $= 822.2121281$ $= 822$ km	M1 A1 **2**	
16.	$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 2 \\ 1 & 1 \end{pmatrix}$ $a + 2b = \frac{1}{2}$ $\underline{4a + 2b = 2}$ $3a = \frac{3}{2} \Rightarrow a = \frac{1}{2}$ $\frac{1}{2} + 2b = \frac{1}{2} \Rightarrow b = 0$ $c + 2d = 1$ $\underline{4c + 2d = 1}$ $3c = 0 \Rightarrow c = 0$ $0 + 2d = 1 \Rightarrow d = \frac{1}{2}$ $\therefore M = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$	M1 M1 A1 **3**	: formation and solution of simultaneous equations : formation and solution of simultaneous equations
17.	(a) (i) $\frac{276000 - 60000}{18}$ $= 12\,000$ (ii) 276000×0.9 $= 248400$ (b) 248400×0.95 $= 235980$ 235980×1.2^2 $= 339811.2$ (c) $339811.2 - 276000$ $\frac{63811.2}{276000} \times 100$ $= 23.12\ \%$	M1 A1 M1 A1 M1 M1 A1 M1 M1 A1 **10**	

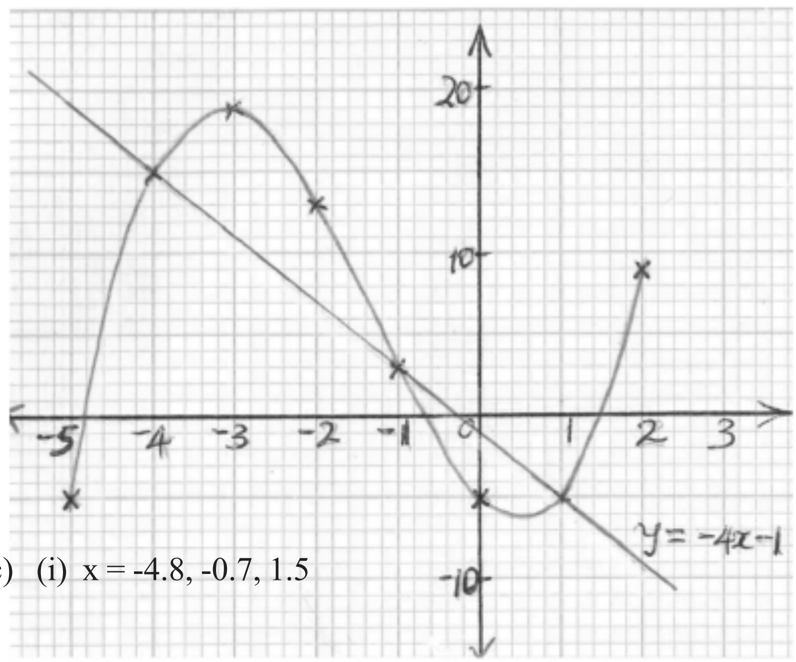
18.	(a) $\angle QPR = 90° - 72° = 18°$ $\angle PQR = 90°$ - angle subtended by diameter	B1 B1	
	(b) $\angle PQS = 180° - 2(72) = 36°$ $\angle PSQ = 72°$ - angle subtended at the circumference by chord PQ equal and base $\angle$'s of isosceles $\Delta QPS = 72°$	B1 B1	
	(c) $\angle OQS = 36° - 18° = 18°$ base angles of isosceles Δ OPQ = 18°	B1 B1	or equivalent
	(d) $\angle RTS = 180 - (36 + 18) = 126°$ extension angle RTS equal to sum of opposite interior angles TSP and TPS	B1 B1	
	(e) $\angle RSV = 90° - 36° = 54°$ $\angle RSV = \angle RPS$ - angle in alternate segment.	B1 B1	
		10	

19. (a)

x	-5	-4	-3	-2	-1	0	1	2
y $=x^3+4x^2-5x-5$	-5	15		13	3		-5	9

B2 — allow B1 for 4 correct

(b)



S1 — Suitable scale

P1 — All correctly plotted

C1

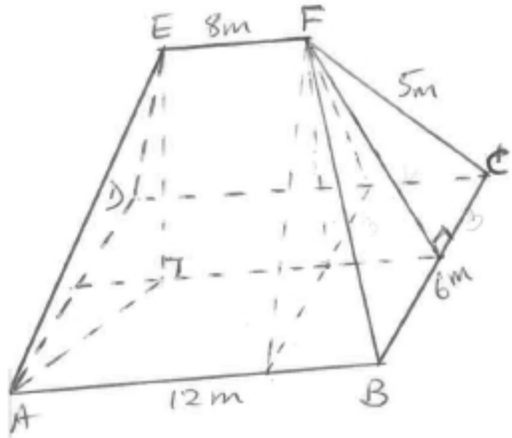
(c) (i) x = -4.8, -0.7, 1.5 — B2 — ±0.1 allow B1 for 2

(ii) y = -4x - 1 — P1 — values : plotting for line

Solutions — L1

x = -4, -1, 1. — B1

10

20.	(a) = distance of EF from place ABCD		
	slant height from F to BC		
	= $\sqrt{5^2 - 3^2}$	M1	
	=4		
	` = distance of EF from plane ABCD		
	= $\sqrt{4^2 - 2^2}$	M1	
	= $\sqrt{12}$ = 3.46 m	A1	
	(b) (i) angle between planes		or equivalent
	ADE and ABCD		
	= $\tan^{-1}\frac{\sqrt{12}}{2}$	M1	
	= 60°	A1	
	(ii) angle between line AE and plane ABCD		
	= $\sin^{-1}\frac{\sqrt{12}}{5}$	M1	or equivalent
	= 43.9°	A1	
	(iii) angle between planes		
	ABFE and DCFE		
	= $2\tan^{-1}\frac{3}{\sqrt{12}}$	M1	$\tan^{-1}\frac{3}{\sqrt{12}}$ or equivalent
	= 81.8°	M1	doubling
		A1	
		10	

21. (a)

x	0	40	80	120	160	200	240
y= 2 sin x + 20		1.7		1.3		-1.3	
y= $\sqrt{3}$ cos x			0.3		-1.6		-0.9

B1
B1

(b)

S1 — suitable scale used

(c) (i) 2 sin (x + 20) = $3 \cos x$

Mark	Comment
P1	plotting 2 Sin (x + 20)
P1	plotting 3 cos x
C1	curve for 2 sin x + 20
C1	curve for $\sqrt{3}$ cos x

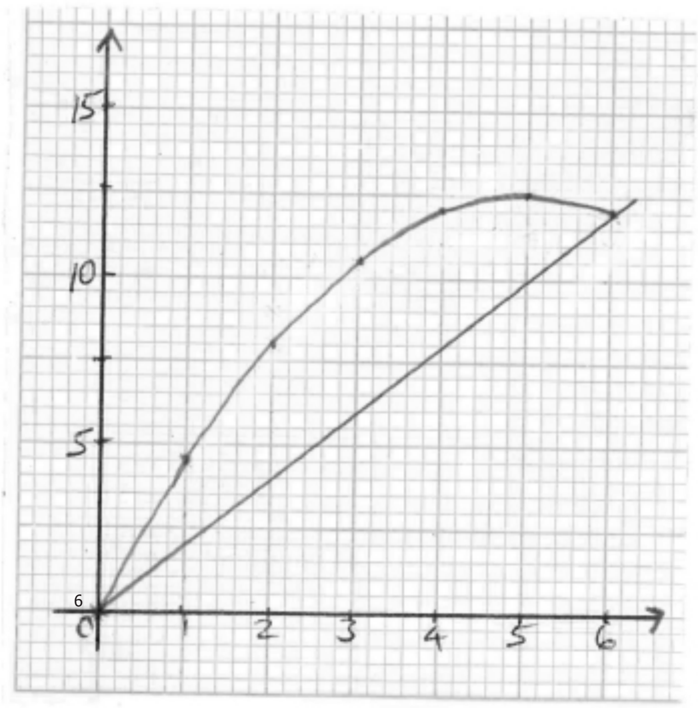
x = 30° — B1
and x = 210° — B1

(ii) amplitude difference

2 - 1.7 = 0.3 — B1

10

22.	(a) $R \propto \frac{S}{T^2} \Rightarrow R = \frac{kS}{T^2}$	B1	
	R = 480 when S = 150 and T = 5		
	$\Rightarrow 480 = \frac{k \times 150}{5^2} = \frac{150k}{25}$	M1	
	$\Rightarrow \quad k = \frac{480 \times 25}{150} = 80$	A1	
	(b) (i) $R = \frac{80S}{T^2}$	B1	
	$R = \frac{80 \times 360}{(1.5)^2} = \frac{80 \times 360}{2.25} = 12800$	M1 A1	
	(ii) $S_2 = 1.05s$, $T_2 = 0.8T$	B1	
	$R_2 = \frac{80 \times 1.05S}{(0.8T)^2} = \frac{80 \times 1.05}{(0.8)^2} \times \frac{S}{T^2}$	M1	
	$R_2 = 131.25\frac{S}{T^2}$		
	$\left(\frac{R_2 - R}{R}\right) \times 100\% = \left(\frac{131.25\frac{S}{T^2} - 80\frac{S}{T^2}}{80\frac{S}{T^2}}\right) \times 100\%$	M1	
	$= \frac{\frac{S}{T^2}}{\frac{S}{T^2}}\left(\frac{131.25 - 80}{80}\right) \times 100$		
	$= 64.0625$	A1	
	$= 64.06\ \%$		
		10	

Q	Working	Marks	Remarks
23.	(a)		
		B1	table may be implied
	(b) 	P1	: plotting
		C1	: curve
	$\int_0^6 \left(5x - \frac{1}{2}x^2\right)dx$	M1	: integral
	$= \left[\frac{5}{2}x^2 - \frac{1}{2 \times 3}x^3\right]_0^6$		
	$= \left(\frac{5 \times 6^2}{2} - \frac{1}{6} \times 6^3\right) - 0$	M1	: substitution
	$= \left(90 - 36\right) - 0 = 54$	A1	
	(c) (i) Drawing line $y = 2x$	L1	
	(ii) Area of Δ : $\frac{1}{2} \times 6 \times 12$	M1	
	$= 36$	A1	
	$\therefore$ Bounded area $= 54 - 36 = 18$	B1	
		10	

x	0	1	2	3	4	5	6
$y = 5x - \frac{1}{2}x^2$	0	4.5	8	10.5	12	12.5	12

24. (a)

Marks	Frequency	cf	
25-34	4	4	
35-44	5	9	
45-54	8	17	
55-64	12	29	
65-74	9	38	
75-84	3	41	
85-94	1	42	

B1 : marks class column

B1 : frequency column

(b) (i) cfs — B1

S1 : scale

(c) (i) Identification of median

P1 : plotting

C1 : curve

B1

= 57.5 ± 0.5 — B1

(ii) Identification of upper quartile mark — B1

= 66.5 ± 0.5 — B1

10