

PAPER 2

P2 Q 1

Simplify $\frac{4}{\sqrt{5} + \sqrt{2}} - \frac{3}{\sqrt{5} - \sqrt{2}}$ (3 marks)

Answer

$$\begin{aligned} & \frac{4}{\sqrt{5} + \sqrt{2}} - \frac{3}{\sqrt{5} - \sqrt{2}} \\ &= \frac{4(\sqrt{5} - \sqrt{2}) - 3\sqrt{5} + \sqrt{2}}{(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})} \\ &= \frac{4\sqrt{5} - 4\sqrt{2} - 3\sqrt{5} - 3\sqrt{2}}{3} \\ &= \frac{\sqrt{5} - 7\sqrt{2}}{3} \end{aligned}$$

P2 Q 2

By correcting each number to one significant figure, approximate the value of 788×0.006 . Hence calculate the percentage error arising from this approximation. (3 marks)

Answer

Writing each number to one significant figure: $788 \approx 800$ and $0.006 = 0.006$

Thus, $800 \times 0.006 = 4.8$

$$\% \text{ error} = \frac{4.8 - (788 \times 0.006)}{788 \times 0.006} \times 100 \%$$

$$= \frac{0.072}{4.728} \times 100\%$$

$$= 1.523\% \text{ or } (1.52284264\%)$$

P2 Q 3

The area of triangle FGH is 21cm^2 . The triangle FGH is transformed using the matrix

$$\begin{pmatrix} 4 & 5 \\ 1 & 2 \end{pmatrix} \text{ Calculate the area of the image of triangle FGH} \quad (2 \text{ Marks})$$

Answer

$$\begin{aligned} \text{Image area} & [(4 \times 2) - (5 \times 1)] \times 21 \\ &= 63 \text{ sq. units} \end{aligned}$$

P2 Q 4

Make s the subject of the formula.

$$w = 3\sqrt{\frac{s+t}{s}}$$

Answer

$$w^3 = \frac{s+t}{s}$$

$$w^3 s = s+t$$

$$w^3 s - s = t$$

$$s = \frac{t}{w^3 - 1}$$

P2 Q 5

Solve the equation $2\log x - \log(x-2) = 2\log 3$

(3mks)

Answer

$$\text{Log}\left(\frac{x^2}{(x-2)}\right) = \log 3^2$$

$$\frac{x^2}{x-2} = 9$$

$$x^2 - 9x + 18 = 0$$

$$(x-6)(x-3) = 0$$

$$x = 6 \text{ or } x = 3$$

P2 Q 6

Kago deposited Ksh 30 000 in a financial institution that paid simple interest at the rate of 12% per annum. Nekesa deposited the same amount of money as Kago in another financial institution that paid compound interest. After 5 years, they had equal amounts of money in the financial institutions.

Determine the compound interest rate per annum, to 1 decimal place, for Nekesa's deposit. (4 marks)

Answer

$$\begin{aligned} \text{Amount for Kago} \\ &= 30000 + \frac{12}{100} \times 30000 \times 5 \\ &= 48000 \end{aligned}$$

$$\begin{aligned} \text{Compound interest rate for Nekesa} \\ 30000\left(1 + \frac{r}{100}\right)^5 = 48000 \end{aligned}$$

$$\left(1 + \frac{r}{100}\right)^5 = \frac{48000}{30000} = 1.6$$

$$1 + \frac{r}{100} = \sqrt[5]{1.6}$$

$$\begin{aligned} r &= 100(1.098560543 - 1) \\ &= 9.9\% \end{aligned}$$

P2 Q7

The masses in kilograms of 20 bags of maize were; 90,94,96,98,99,102,105 91 102 99 105,94,99,90,94,99,98,96,102 and 105.

Using an assumed mean of 96kg, calculate the mean mass, per bag, of the maize. (3 marks)

Answer

Differences from assumed mean

$$\begin{aligned} & -6 - 2 + 0 + 2 + 3 + 6 + 9 - 5 + 6 + 3 + 9 \\ & -2 + 3 - 6 - 2 + 3 + 2 + 0 + 6 + 9 = 38 \end{aligned}$$

$$\begin{aligned} \therefore \text{mean} &= 96 + \frac{38}{20} \\ &= 97.9 \end{aligned}$$

P2 Q 8

The first term of an arithmetic sequence is - 7 and the common difference is 3.

(a)List the first six terms of the sequence; (1 mark)

Answer

First 6 terms

$$-7, -4, -1, 2, 5, 8$$

(b)Determine the sum of the first 50 terms of the sequence. (2 marks)

Answer

Sum of 1st 50 terms

$$S_{50} = \frac{50}{2} \{2 \times -7 + 49 \times 3\}$$

$$= 3325$$

P2 Q 9

A bag contains 2 white balls and 3 black balls. A second bag contains 3 white balls and 2 black balls. The balls are identical except for the colours.

Two balls are drawn at random, one after the other from the first bag and placed in the second bag. Calculate the probability that the 2 balls are both white. (2 marks)

Answer

$$P(ww) = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$$

$$P(bb) = \frac{2}{7} \times \frac{1}{6} = \frac{1}{21}$$

P2 Q. 10

An arc 11 cm long, subtends an angle of 70° at the centre of a circle.

Calculate the length, correct to one decimal place, of a chord that subtends an angle of 90° at the centre of the same circle. (4 marks)

Answer

Length 12.7 cm

P2 Q 11

Given that $q \mathbf{i} + \frac{1}{3} \mathbf{j} + \frac{2}{3} \mathbf{k}$ is a unit vector, find q (2 marks)

Answer

$$q^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = 1^2$$

$$q^2 = 1 - \frac{5}{9} = \frac{4}{9}$$

$$q = \pm \frac{2}{3}$$

P2 Q 12

(a) Expand the expression $(1 + \frac{1}{2}x)^5$ in ascending powers of x , leaving the coefficients as fractions in their simplest form (2 marks)

Answer

$$\begin{aligned} & 1.1^5 (\frac{1}{2}x)^0 + 5.1^4 (\frac{1}{2}x)^1 + 10.1^3 (\frac{1}{2}x)^2 + 10.1^2 (\frac{1}{2}x)^3 + 5.1 (\frac{1}{2}x)^4 + 1.1^0 (\frac{1}{2}x)^5 \\ & = 1 + \frac{5}{2}x + \frac{5}{2}x^2 + \frac{5}{4}x^3 + \frac{5}{16}x^4 + \frac{1}{32}x^5 \end{aligned}$$

(b) Use the first three terms of the expansion in (a) above to estimate the value of $(1 \frac{1}{20})^5$ (2 marks)

Answer

$$\begin{aligned} \left(1 \frac{1}{20}\right)^5 &= 1 + \frac{5}{2} \times \frac{1}{10} + \frac{5}{2} \times \frac{1}{100} \\ &= 1 + \frac{5}{20} + \frac{5}{200} \\ &= 1 \frac{11}{40} \end{aligned}$$

P2 Q 13

A circle whose equation is $(x-1)^2 + (y-k)^2 = 10$ passes through the point $(2, 5)$, Find the coordinates of the two possible centres of the circle,

Answer

$$(2-1)^2 + (5-k)^2 = 10$$

$$k^2 - 10k + 16 = 0$$

$$(k-2)(k-8) = 0$$

$$k = 2 \text{ or } k = 8$$

Centre at $(1,2)$ or $(1,8)$

P2 Q 14

Water and milk are mixed such that the ratio of the volume of water to that of milk is 4: 1. Taking the density of water as 1 g/cm³ and that of milk as 1.2g/cm³, find the mass in grams of 2.5 litres of the mixture. (3 marks)

Answer

$$\text{Vol. of water} = \frac{4}{5} \times 2500 = 2000 \text{ cm}^3$$

$$\text{Vol. of milk} = \frac{1}{5} \times 2500 = 500 \text{ cm}^3$$

$$\begin{aligned} \text{The mass of the mixture} &= 2000 \times 1\text{g} + 500 \times 1.2\text{g} \\ &= 2600\text{g} \end{aligned}$$

PP2 No. 15]

A school decided to buy at least 32 bags of maize and beans. The number of bags of maize were to be more than 20 and the number of bags of beans were to be at least 6. A bag of maize costs Ksh 2500 and a bag of beans costs Ksh 3500. The school had Ksh 100 000 to purchase the maize and beans. Write down all the inequalities that satisfy the above information. (4 marks)

Answer

$$x+y \geq 32, x > 20, y \geq 6, 5x + 7y \leq 200$$

P2 Q 16

Find in radians, the values of x in the interval $0 \leq x \leq 2\pi$ for which $2 \cos^2 x - \sin x = 1$. (Leave the answers in terms of π) (4mks)

Answer

$$2(1 - \sin^2 x) - \sin x = 1$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$2\sin^2 x + 2\sin x - \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2} \text{ or } \sin x = -1$$

$$x = \frac{1}{6}\pi, \frac{5}{6}\pi, \frac{3}{2}\pi$$

(b) Calculate the mid-ordinates for 5 strips between $x = 1$ and $x = 6$, and hence use the mid-ordinate rule to approximate the area under the curve between $x = 1$, $x = 6$ and the x -axis. (3 marks)

Answer

$$y_1 = \frac{1}{2} \times 1.5^2 - 1.5 + 3 = 2.625$$

$$y_2 = \frac{1}{2} \times 2.5^2 - 2.5 + 3 = 3.625$$

$$y_3 = \frac{1}{2} \times 3.5^2 - 3.5 + 3 = 5.625$$

$$y_4 = \frac{1}{2} \times 4.5^2 - 4.5 + 3 = 8.625$$

$$y_5 = \frac{1}{2} \times 5.5^2 - 5.5 + 3 = 12.625$$

Approximate area

$$= \frac{1}{2}(2.625 + 3.625 + 5.625 + 8.625 + 12.625)$$

$$= 33.125 \text{ sq. units}$$

(c) Assuming that the area determined by integration to be the actual area, calculate the percentage error in using the mid-ordinate rule. (4 marks)

Answer

$$\begin{aligned} \text{Area} &= \int_1^6 \left(\frac{1}{2}x^2 - x + 3 \right) dx = \left[\frac{x^3}{6} - \frac{x^2}{2} + 3x \right]_1^6 \\ &= \left[\frac{6^3}{6} - \frac{6^2}{2} + 3 \times 6 \right] - \left[\frac{1^3}{6} - \frac{1^2}{2} + 3 \right] = 33.5 \\ \% \text{ error} &= \frac{33.5 - 33.125}{33.5} \times 100 \\ &= 0.625\% \end{aligned}$$

P2 Q 17

A garden measures 10 m long and 8 m wide. A path of uniform width is made all round the garden. The total area of the garden and the path is 168 m².

(a) Find the width of the path. (4 marks)

Answer

Let width of the path be x

$$\text{Area} = (10 + 2x)(8 + 2x) = 168$$

$$\Leftrightarrow 80 + 20x + 16x + 4x^2 = 168$$

$$4x^2 + 36x - 88 = 0$$

$$\Leftrightarrow x^2 + 9x - 22 = 0$$

$$(x - 2)(x + 11) = 0$$

$$x = 2 \text{ or } -11$$

Width of path = 2m

(b) The path is to be covered with square concrete slabs. Each corner of the path is covered with a slab whose side is equal to the width of the path. The rest of the path is covered with slabs of side 50 cm. The cost of making each corner slab is sh 600 while the cost of making each smaller slab is sh 50. Calculate:

(i) the number of the smaller slabs used.

(3 marks)

Answer

Area covered by small slabs

$$= \frac{14}{68} \times 12 - (10 \times 8 + 4(2 \times 2))$$

$$\begin{aligned} \text{No. of slabs} &= \frac{72}{0.5 \times 0.5} \\ &= 288 \end{aligned}$$

(ii) the total cost of the slabs used to cover the whole path.

(3 marks)

Answer

Cost of slabs

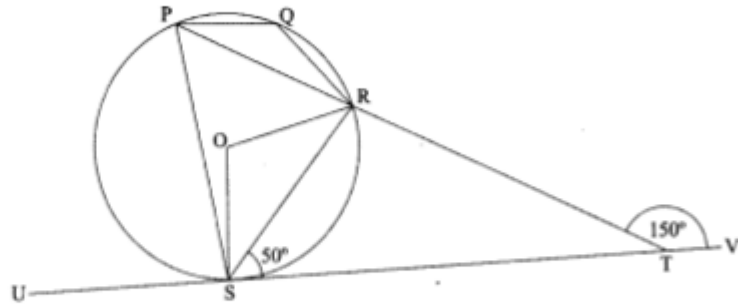
$$\begin{aligned} \text{Large} &= 600 \times 4 \\ &= 2400 \end{aligned}$$

$$\begin{aligned} \text{Small} &= 50 \times 288 \\ &= 14400 \end{aligned}$$

$$\begin{aligned} \text{Total cost} &= 2400 + 14400 \\ &= \text{sh}16,800 \end{aligned}$$

P2 Q 18

In the figure below, P, Q, R and S are points on the circle centre O. PRT and USTV are straight lines. Line USTV is a tangent to the circle at S, $\angle RST = 50^\circ$ and $\angle RTV = 150^\circ$.



a) Calculate the size of:

(i) $\angle ORS$; (2 marks)

Answer

$$\angle OSR = 90 - 50 = 40$$

$$\therefore \angle ORS = 40$$

(ii) $\angle USP$; (1 mark)

Answer

$$\angle RTS = 30^\circ \text{ and } \angle RPS = 50^\circ$$

$$\therefore \angle USP = 30^\circ + 50^\circ = 80^\circ$$

(iii) $\angle PQR$. (2 marks)

Answer

$$\angle PQR = 180^\circ - 50^\circ = 130^\circ$$

(b) Given that $RT = 7$ cm and $ST = 9$ cm, calculate to 3 significant figures:

(i) the length of line PR; (2 marks)

Answer

$$PT \times TR = TS^2$$

$$(7 + x)(7) = 9^2$$

$$7x = 81 - 49 = 32$$

$$x = \frac{32}{7} = 4.57$$

(ii) the radius of the circle. (3 marks)

Answer

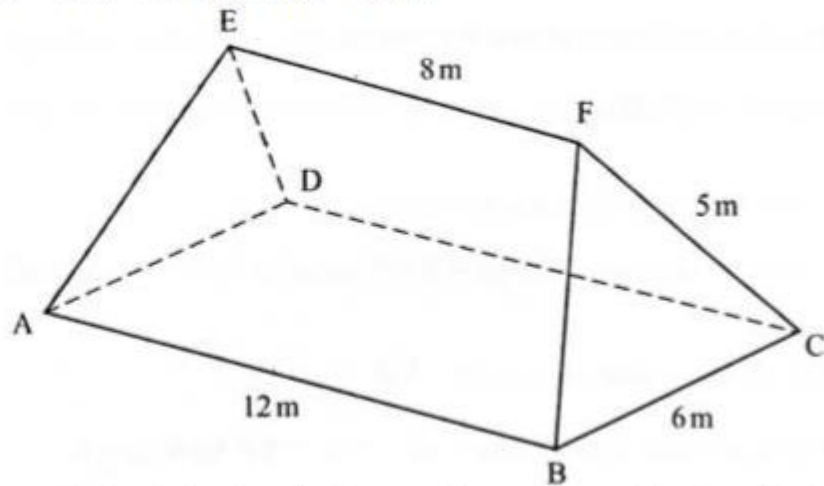
$$\angle ORP = 40^\circ$$

$$\cos 40^\circ = \frac{\frac{1}{2} \times 4.57}{r}$$

$$r = \frac{\frac{1}{2} \times 4.57}{\cos 40} = 2.98$$

P2 Q 19

The figure ABCDEF below represents a roof of a house. $AB = DC = 12$ m, $BC = AD = 6$ m, $AE = BF = CF = DE = 5$ m and $EF = 8$ m.



(a) Calculate, correct to 2 decimal places, the perpendicular distance of EF from the plane ABCD. (3 marks)

Answer

(a) = distance of EF from plane ABCD

slant height from F to BC

$$= \sqrt{5^2 - 3^2}$$

$$= 4$$

\therefore = distance of EF from plane ABCD

$$= \sqrt{4^2 - 2^2}$$

$$= \sqrt{12} = 3.46 \text{ m}$$

(b) Calculate the angle between:

(i) the planes ADE and ABCD; (2 marks)

Answer

(i) angle between planes

ADE and ABCD

$$= \tan^{-1} \frac{\sqrt{12}}{2}$$

$$= 60^\circ$$

(ii) the line AE and the plane ABCD, correct to 1 decimal place; (2 marks)

Answer

∴ angle between line AE
and plane ABCD

$$= \sin^{-1} \frac{\sqrt{12}}{5}$$

$$= 43.9^\circ$$

(iii) the planes ABFE and DCFE, correct to 1 decimal place. (3 marks)

Answer

angle between planes

ABFE and DCFE

$$= 2 \tan^{-1} \frac{3}{\sqrt{12}} \text{ m}$$

$$= 81.8^\circ$$

P2 Q 20

A water vendor has a tank of capacity 18 900 litres. The tank is being filled with water from two pipes A and B which are closed immediately when the tank is full. Water flows at the rate of 150 000cm³/minute through pipe A and 120 000 cm³/minute through pipe B.

a) If the tank is empty and the two pipes are opened at the same time, calculate the time it takes to fill the tank (3 marks)

Answer

Total rate of flow in litres

$$\therefore 120 + 150 = 270 \text{ l/min}$$

$$\text{Time taken} = \frac{18900}{270}$$

$$= 70 \text{ min (1 hr 10 min)}$$

b) On a certain day the vendor opened the two pipes A and B to fill the empty tank. After 25 minutes he opened the outlet tap to supply water to his customers at an average rate of 20 litres per minute.

i) Calculate the time it took to fill the tank on that day (4 marks)

Answer

Part of tank filled after 25 min

$$\therefore 270 \times 25$$

$$= 6750$$

Time taken to fill remaining part

$$= \frac{18900 - 6750}{270 - 20}$$

$$= 48.6 \text{ min}$$

$$\text{Total time to fill tank}$$

$$= 25 + 48.6 = 73.6 \text{ min}$$

ii) The vendor supplied a total of 542 jerricans, each containing 25 litres of water, on that day. If the water that remained in the tank was 6300 litres, calculate, in litres, the amount of water that was wasted (3 marks)

Answer

$$\begin{aligned} \text{Total inflow into tank} \\ &= 270 \times 73.6 \\ &= 19872 \\ \text{Water wasted} &= 19872 - (542 \times 25 + 6300) \\ &= 221 \end{aligned}$$

P2 Q 21

A tourist took 1h 20minutes to travel by an aircraft from town T(3°S, 35°E) to town U(9°N,35°E).

(Take the radius of the earth to be 6370km and $(\pi = \frac{22}{7})$)

(a) Find the average speed of the aircraft.(3 marks)

Answer

$$\begin{aligned} \text{distance from T to U} \\ &= 2 \times 6370 \times \frac{22}{7} \times \frac{12}{360} \\ \text{speed} &= \frac{2 \times 6370 \times \frac{22}{7} \times \frac{12}{360}}{1\frac{1}{3}} \\ &= 1001 \text{ km/h} \end{aligned}$$

(b)After staying at town U for 30 minutes, the tourist took a second aircraft to town V(9°N, 5°E). The average speed of the second aircraft was 90% that of the first

aircraft. Determine the time, to the nearest minute, the aircraft took to travel from U to V. (3 marks)

Answer

$$\begin{aligned} \text{time} &= \frac{2 \times 6370 \times \frac{22}{7} \times \frac{30}{360} \cos 9^\circ}{1001 \times \frac{90}{100}} \\ &= 3.658104965 \text{ h} \\ &\approx 3 \text{ h } 39 \text{ min} \end{aligned}$$

(c) When the journey started at town T, the local time was 0700h. Find the local time at V when the tourist arrived. (4 marks)

Answer

$$\begin{aligned} \text{Arrival time at U} \\ 0700 + 1\text{h } 20\text{ min} \\ = 0820\text{ h} \end{aligned}$$

$$\begin{aligned} \text{Departure time at U} \\ 0820 + 30\text{ min} \\ = 0850\text{ h} \end{aligned}$$

$$\begin{aligned} \text{Time difference between U and V} \\ \frac{35 - 5}{360} \times 24 \\ = 2\text{h} \end{aligned}$$

$$\begin{aligned} \text{Arrival time at V (local time)} \\ 0850\text{h} + 3\text{h } 39\text{min} - 2\text{h} \\ = 1029\text{h} \end{aligned}$$

P2 Q 22

The gradient function of a curve is given by the expression $2x + 1$. If the curve passes through the point $(-4, 6)$;

(a) Find:

(i) The equation of the curve (3 marks)

Answer

$$\begin{aligned} y &= \frac{2x^2}{2} + x + c \\ \text{at } x &= -4, y = 6 \\ 6 &= (-4)^2 - 4 + c \\ c &= -6 \\ y &= x^2 + x - 6 \end{aligned}$$

(ii) The values of x , at which the curve cuts the x -axis (3 marks)

Answer

$$\begin{aligned} x^2 + x - 6 &= 0 \\ (x - 2)(x + 3) &= 0 \\ x &= 2 \text{ or } x = -3 \end{aligned}$$

(b) Determine the area enclosed by the curve and the x -axis (4 marks)

Answer

$$\begin{aligned} \int_{-3}^2 (x^2 + x - 6) dx &= \left[\frac{x^3}{3} + \frac{x^2}{2} - 6x \right]_{-3}^2 \\ \left[\frac{8}{3} + \frac{4}{2} - 12 \right] - \left[\frac{-27}{3} + \frac{9}{2} + 18 \right] \\ -7\frac{1}{3} - 13\frac{1}{2} &= -20\frac{5}{6} \\ \text{Area} &= 20\frac{5}{6} \end{aligned}$$

P2 Q 23

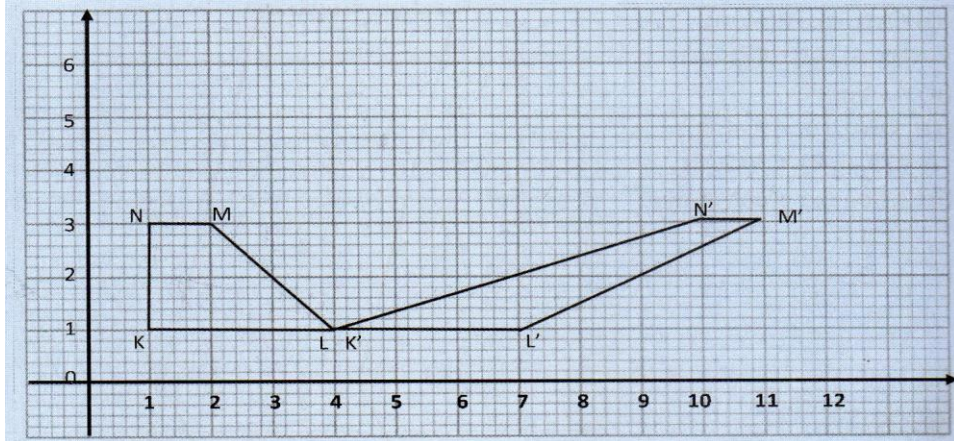
A quadrilateral with vertices at $K(1, 1)$, $L(4, 1)$, $M(2, 3)$ and $N(1, 3)$ is transformed by a matrix $T = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ to a quadrilateral $K'L'M'N'$

(a) Determine the coordinates of the image. (3 marks)

Answer

$K'(4, 1)$, $L'(7, 1)$, $M'(11, 3)$, $N'(10, 3)$

(b) On the grid provided draw the object and the image. (2 marks)



(c)(i) Describe fully the transformation which maps $KLMN$ onto $K'L'M'N'$. (2 marks)

Answer

Shear parallel to x -axis of shear factor 3 with the x -axis as the invariant line

(ii) Determine the area of the image. (1 mark)

Answer

Area of image = Area of object = 4 sq units

(d) Find a matrix which maps $K'L'M'N'$ onto $KLMN$. (2 marks)

Answer

$$\begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}$$

P2 Q 24

The first, fifth and seventh terms of an arithmetic progression (AP) correspond to the first three consecutive terms of a decreasing Geometric Progression (GP). The first term of each progression is 64, and the common difference of the AP is d and the common ratio of the G.P is r .

a) (i) Write two equations involving d and r (2marks)

Answer

$$\begin{aligned} 64 + 4d &= 64r \\ 64 + 6d &= 64r^2 \end{aligned}$$

(ii) Find the values of d and r

Answer

From (i)

$$d = 16r - 16$$

$$64r^2 = 64 + 6(16r - 16)$$

$$64r^2 = 64 + 96r - 96$$

$$2r^2 - 3r + 1 = 0$$

$$(2r - 1)(r - 1) = 0$$

$$r = \frac{1}{2} \text{ or } r = 1$$

For decreasing GP, $r = \frac{1}{2}$

Substituting $r = \frac{1}{2}$ in (i)

$$64 \times \frac{1}{2} = 64 + 4d$$

$$d = -8$$

b) Find the sum of the first 10 term of;

(i) the A.P

Answer

A.P

$$S_{10} = \frac{10}{2} \{ 2 \times 64 + 9 \times -8 \} = 280$$

(ii) the G.P

Answer

G.P

$$S_{10} = \frac{64 \left(1 - \frac{1}{2^{10}} \right)}{1 - \frac{1}{2}} = \frac{64 \left(\frac{1023}{1024} \right)}{\frac{1}{2}} = 127.875$$