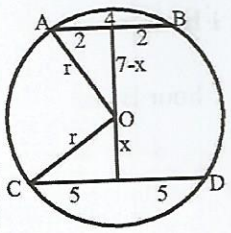
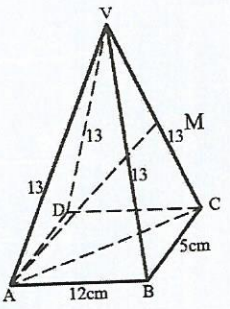
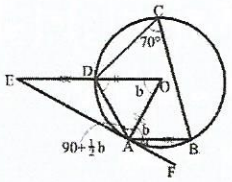
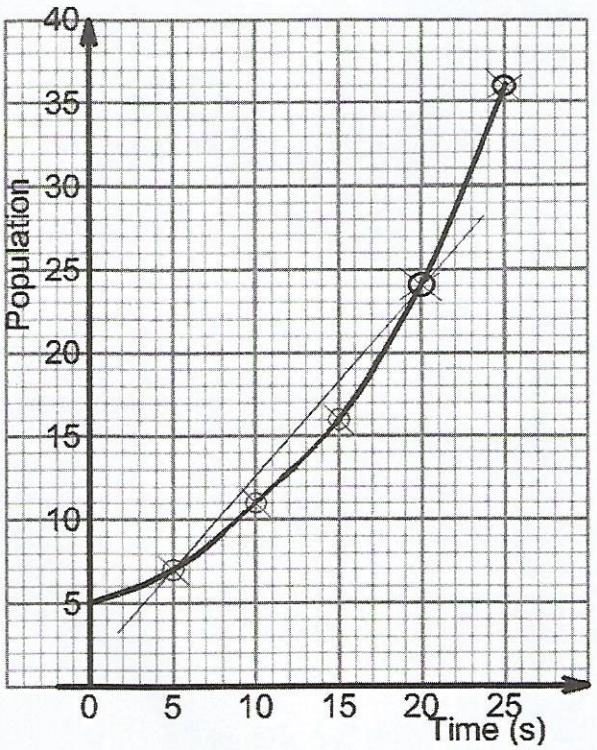
KCSE 2022 PAPER 2

5.1.2 Mathematics Alt A Paper 2 (121/2)

No.	Working	Marks	Comments
1.	$1500 + 1450 + \ldots + 50$ $50 = 1500 + (n-1)\times -50$ $n-1 = \frac{1450}{50}$ $n = 30$ $S_{30} = \frac{30}{2}\{3000 + 29\times -50\}$ $= 15\times 1550$ $= 23250$	B1 M1 A1 **3**	
2.	In 1 hour, machine A does $\frac{1}{8}$ of the task In 1 hour, machines A and C do $\frac{1}{5}$ of the task In 1 hour, machine C does $\frac{1}{5} - \frac{1}{8} = \frac{3}{40}$ of the task In 1 hour B and C do $\frac{1}{10} + \frac{3}{40} = \frac{4+3}{40} = \frac{7}{40}$ of the task $\therefore$ Machines B and C would take $\frac{40}{7} = 5\frac{5}{7}$ hours	M1 M1 A1 **3**	
3.	$\frac{3+\sqrt{5}}{7-3\sqrt{5}} = \frac{(3+\sqrt{5})(7+3\sqrt{5})}{(7-3\sqrt{5})(7+3\sqrt{5})}$ $= \frac{21 + 9\sqrt{5} + 7\sqrt{5} + 3\times 5}{49 - 9\times 5}$ $= \frac{36 + 16\sqrt{5}}{4}$ $= 9 + 4\sqrt{5}$	M1 A1 **2**	

No.	Working	Marks	Comments
4.	$V = k \times m^2$ $600\,000 = 100\ k$ $k = 6\,000$ Value of a similar stone of mass 18.5 kg : $= 6\,000 \times 18.5^2$ = Ksh 2 053 500	B1 M1 A1 **3**	V = Value m = Mass
5.	$2x + 2y = 48 \Rightarrow x + y = 24$ (i) $xy = 108 \Rightarrow$ (ii) $x(24 - x) = 108$ $x^2 - 24x + 108 = 0$ $(x-6)(x-18) = 0$ $x = 18,\ y = 6$	M1 M1 A1 **3**	x = length y = width
6.	Let radius = r and distance of O from CD = x $x^2 + 5^2 = r^2$ $(7-x)^2 + 2^2 = r^2$ $x^2 + 25 = 49 - 14x + x^2 + 4$ $14x = 28$ $x = 2$ $r^2 = 4 + 25$ $r = \sqrt{29}$	M1 A1 B1 **3**	
7.	AC = 13 cm Δ AVC is equilateral of side 13 cm $CM = \frac{13}{2} = 6.5$ cm $\therefore AM^2 = \sqrt{13^2 - 6.5^2}$ = 11.26 cm	B1 M1 A1 **3**	

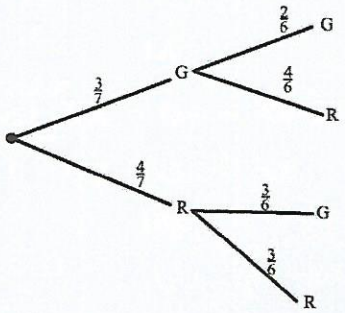
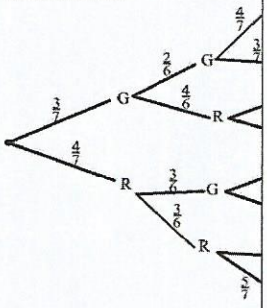
No.	Working	Marks	Comments
8.	$b = 40°$ $a = 50$	M1 A1 M1 A1 4	 $\angle DAC = 90 - \frac{1}{2}b$ B1 Base $\angle$s of isosceles Δ $90° - \frac{1}{2}b + b + 70° = 180$ M1 opp. cyclic quad $\therefore \frac{1}{2}b = 180 - 160$ $\Rightarrow b = 40°$ A1 $a = 90° - 40$ $= 50°$ B1
9.	(a)  (b) Average rate of change $= \frac{24 - 7}{20 - 5}$ $= 1.13$	P1 C1 M1 A1 4	

No.	Working	Marks	Comments
10.	$\mathbf{AC} = \begin{pmatrix}5\\5\end{pmatrix} - \begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}4\\2\end{pmatrix}$ $r^2 = 4^2 + 2^2$ $r = \sqrt{20}$ $(x-5)^2 + (y-5)^2 = 20$ when $y = 9, x = a$ $\therefore (a-5)^2 + (9-5)^2 = 20$ $(a-5)^2 = 20-16 = 4$ $a-5 = \pm 2$ $a = 7$ or 3	B1 M1 A1 **3**	$\mathbf{CB} = \begin{pmatrix}a\\9\end{pmatrix} - \begin{pmatrix}5\\5\end{pmatrix}$ $= \begin{pmatrix}a-5\\4\end{pmatrix}$ $(a-5)^2 + 4^2 = \left(\sqrt{20}\right)^2$
11.	Amplitude, A = 2 Let Period = w $\frac{360}{w} = 60$ $w = 6$	B1 M1 A1 **3**	
12.	$\bar{x} = \frac{5+6+8+9+13+14+15}{7} = 10$ $\text{Variance} = \sqrt{\frac{(-5)^2+(-4)^2+(-2)^2+(-1)^2+3^2+4^2+5^2}{7}}$ $= \frac{96}{7} = 13.71$ $\text{s.d} = \sqrt{13.71}$ $= 3.703$ ≈ 4	B1 M1 A1 **3**	
13.	1^{st} slab: $= 12298 \times \frac{10}{100} = 1229.8$ 2^{nd} slab: $= 11587 \times \frac{15}{100} = 1738.05$ 3^{rd} slab $= 2660.75 \times \frac{20}{100} = 532.15$ Total $= 3500$ Tax after relief $= 3500 - 1408$ $=$ Ksh 2 092	M1 M1 A1 **3**	

No.	Working	Marks	Comments
14.	Let B =(x, y, z) $\begin{pmatrix}8\\4\\-1\end{pmatrix} = \frac{1}{5}\begin{pmatrix}4\\8\\3\end{pmatrix} + \frac{4}{5}\begin{pmatrix}x\\y\\z\end{pmatrix}$ $\frac{4}{5}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}8+0.8\\4-1.6\\-1-0.6\end{pmatrix} = \begin{pmatrix}8.8\\2.4\\-1.6\end{pmatrix}$ $\begin{pmatrix}x\\y\\z\end{pmatrix} = \frac{5}{4}\begin{pmatrix}8.8\\2.4\\-1.6\end{pmatrix} = \begin{pmatrix}11\\3\\-2\end{pmatrix}$ $B(11, 3, -2)$	M1 M1 A1 **3**	Follow through
15.	Longitude difference = 65° − 40° = 25° Time difference = $25\times4 = 1\frac{2}{3}$h distance AB = 25×60 = 1500 nm Time of flight = 1340 − 1100 = $2\frac{2}{3}$h Speed = $\frac{1500\text{ nm}}{2\frac{2}{3}}$ = 562.5 knots	M1 M1 M1 A1 **4**	Time at A when it is 1200h at B 1200 + 0140 1340 h
16.	$\int_0^1(-2t+4)\,dt$ $= \left[-\frac{2t^2}{2}+4t\right]_0^1 = \left[-t^2+4t\right]_0^1$ $= (-1+4)-(0)$ = 3 m	M1 M1 A1 **3**	

SECTION B

No.	Mathematics ALT A paper 2	Marks	Comments
17.	(a) (i) In 10 kg of blend A:		
	$\text{Refu}: \frac{3}{5}\times 10 = 6\,\text{kg}$ $\text{Tamu}: \frac{2}{5}\times 10 = 4\,\text{kg}$	B1	Either √
	Let x be kgs of Tamu added:	M1	
	$\frac{6}{4+x} = \frac{1}{2}$	A1	
	$x = 8$		
	(ii) 1 kg of Refu mixed with 2 kg of Tamu		
	$\text{Wholesale price} = \frac{1\times 80 + 2\times 140}{3}$	M1	
	$= 120$	M1	
	$\text{Retailer's price} = \frac{120}{100}\times 120$		
	$= 144$	A1	
	(b) In x kg of Blend A		
	$\text{Refu} = \frac{3}{5}x$, $\text{Tamu} = \frac{2}{5}x$		
	In blend B:		
	$\text{Refu} = \frac{3}{5}x\,\text{kg}$; $\text{Tamu} = \frac{2}{5}x + y\,\text{kg}$	B1	
	Cost of 1 kg of Blend B		
	$\frac{80\times\frac{3}{5}x + 140\times\left(\frac{2}{5}x + y\right)}{x+y} = 130$	M1	
	$48x + 56x + 140y = 130x + 130y$		
	$104x - 130x = 130y - 140y$		
	$-26x = -10y$		
	$\frac{x}{y} = \frac{5}{13}$	A1	
	$\therefore$ Ratio $x:y = 5:13$	B1	
		10	

No.	Mathematics ALT A paper 2	Marks	Comments
18.	(a) $P(R) = \frac{4}{7}$	B1	
	(b) (i) 	B1	
	(ii) $P(\text{Marbles picked were of same colour})$		
	$= \frac{3}{7}\times\frac{2}{6} + \frac{4}{7}\times\frac{3}{6}$	M1	
	$= \frac{18}{42}$	A1	$\left(\text{Accept } \frac{3}{7}\right)$
	(iii) P(at least one red)		
	$= 1 - P(GG)$	M1	
	$= 1 - \frac{3}{7}\times\frac{2}{6}$		
	$= \frac{36}{42}$	A1	
	(c) (i) 7 seen in third branch	B1	For 3rd set of branches√
	$P(\text{Green marble})$		
	$= \frac{3}{7}\times\frac{2}{6}\times\frac{4}{7} + \frac{3}{7}\times\frac{4}{6}\times\frac{3}{7} + \frac{4}{7}\times\frac{3}{6}\times\frac{3}{7} + \frac{4}{7}\times\frac{3}{7}\times\frac{2}{7}$	M1	
	$= \frac{4}{49} + \frac{6}{49} + \frac{6}{49} + \frac{4}{49}$		
	$= \frac{20}{49}$	A1	
	(ii) $P(R) = 1 - \frac{20}{49} = \frac{29}{49}$	B1	
		10	

No.	Mathematics ALT A paper 2	Marks	Comments
19.	(a) (i) Single matrix to map Δ ABC onto Δ A"B"C"		ALT
	$T_2T_1 = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} \times \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix}$	M1	Let co-ordinates of C
	$= \begin{pmatrix} 4.5 & -4 \\ 3 & -2 \end{pmatrix}$	A1	$= (x, y)$
	Single matrix that would map ΔABC onto Δ A"B"C"		$\begin{pmatrix} 4.5 & -4 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 8 \end{pmatrix}$
	Procedure of getting $(T_2T_1)^{-1}$	M1	$\left.\begin{matrix} 4.5x - 4y = 10 \\ 3x - 2y = 8 \end{matrix}\right\}$ M1
	$\left.\begin{matrix} \det = (-9) - (-12) = 3 \\ \therefore (T_2T_1)^{-1} = \frac{1}{3}\begin{pmatrix} -2 & 4 \\ -3 & -4.5 \end{pmatrix} \end{matrix}\right\}$		$1.5x = 6$ M1
			$x = 4$
			$12 - 2y = 8$
			$2y = 4$
			$y = 2$
	Coordinates of C $= \frac{1}{3}\begin{pmatrix} -2 & 4 \\ -3 & 4.5 \end{pmatrix}\begin{pmatrix} 10 \\ 8 \end{pmatrix}$	M1	C = (4,2) A1
	$= \frac{1}{3}\begin{pmatrix} 12 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$		
	Coordinates of C $= (4, 2)$	A1	
	(ii) $\det(T_2T_1) = 3$	B1	
	$\frac{15}{\text{Area } \Delta \text{ ABC}} = 3$	M1	
	Area of Δ ABC $= \frac{15}{3} = 5$ sq units	A1	
	(b) $\begin{pmatrix} 4.5 & -4 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6x + 1 \\ 8 \end{pmatrix}$		
	$4.5x + 4 = 6x + 1$ (i)		Equations (i) & (ii)
	$3 = 1.5x$	M1	
	$x = 2$		
	$3x - 2y = 8$ (ii)		
	$6 - 2y = 8$	A1	
	$y = -1$		
		10	

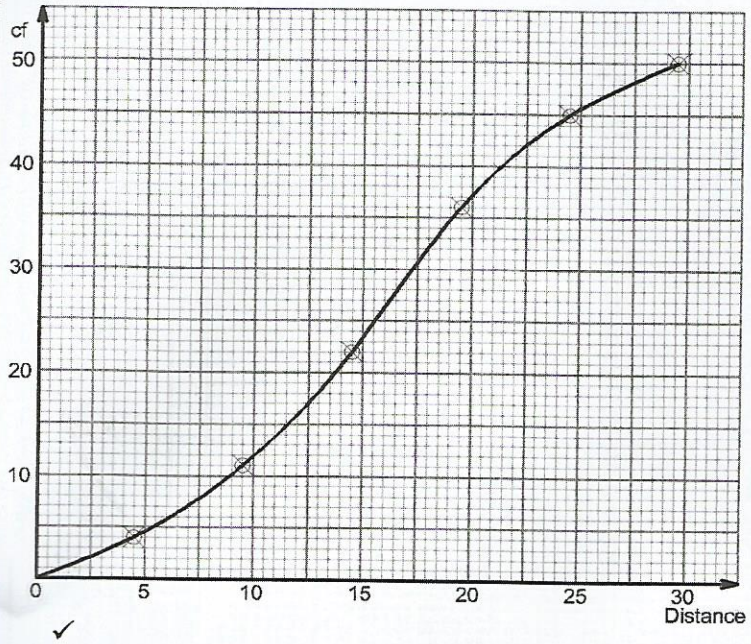
No.	Mathematics ALT A paper 2	Marks	Comments
20.	(a) (i) $\mathbf{AQ} = \begin{pmatrix}9\\1.5\end{pmatrix} - \begin{pmatrix}0\\3\end{pmatrix} = \begin{pmatrix}9\\-1.5\end{pmatrix}$	B1	
	(ii) $\mathbf{CP} = \begin{pmatrix}3\\5\end{pmatrix} - \begin{pmatrix}10.5\\0\end{pmatrix} = \begin{pmatrix}-7.5\\5\end{pmatrix}$	B1	
	(b) (i) $\mathbf{OX} = \mathbf{OA} + \mathbf{AX}$ or $\mathbf{OX} = \mathbf{OC} + \mathbf{CX}$		
	$\mathbf{OX} = \mathbf{OA} + \mathbf{AX}$		
	$= \begin{pmatrix}0\\3\end{pmatrix} + m\begin{pmatrix}9\\-1.5\end{pmatrix}$	B1	
	$= \begin{pmatrix}9m\\3 - 1.5m\end{pmatrix}$		
	$\mathbf{OX} = \mathbf{OC} + \mathbf{CX}$		
	$= \begin{pmatrix}10.5\\0\end{pmatrix} + k\begin{pmatrix}-7.5\\5\end{pmatrix}$	B1	
	$\begin{pmatrix}9m\\3 - 1.5m\end{pmatrix} = \begin{pmatrix}10.5 - 7.5k\\5k\end{pmatrix}$		
	$9m = 10.5 - 7.5k$ or $6m + 5k = 7$ $5k = 3 - 1.5m$ or $3m + 10k = 6$	M1	
	$-\begin{cases}6m + 20k = 12\\6m + 5k = 7\end{cases}$		
	$15k = 5$	M1	
	$k = \frac{1}{3}$	A1	
	$3m + \frac{10}{3} = 6 \Rightarrow m = \frac{8}{9}$	B1	
	(ii) $\mathbf{OX} = \begin{pmatrix}0\\3\end{pmatrix} + \frac{8}{9}\begin{pmatrix}9\\1.5\end{pmatrix}$	M1	
	$= \begin{pmatrix}8\\1\frac{2}{3}\end{pmatrix}$		
	Coordinates of X: $\left(8, 1\frac{2}{3}\right)$	A1	
		10	

No.	Mathematics ALT A paper 2	Marks	Comments
21.	(a) Let the rate of appreciation = r % p.a (House)		
	$3700000 = 2500000\left(1 + \frac{r}{100}\right)^4$	M1	$V_n = V_o\left(1 + \frac{r}{100}\right)^n$
	$\left(1 + \frac{r}{100}\right) = \sqrt[4]{1.48}$	M1	
	$1 + \frac{r}{100} = 1.103$		
	$\frac{r}{100} = 0.1030$	A1	
	$r = 10.30\%$		
	(b) Current value of car		12 seen
	$= 5100000\left(1 - \frac{2}{100}\right)^{12}$	B1 M1	
	$= 4\,002\,055$	A1	
	(c) Let n be number of years		
	$2\,500\,000\left(1 + \frac{10.3}{100}\right)^n = 5\,100\,000\left(1 - \frac{2}{100}\right)^{3n}$	M1	
	$2.5(1.103)^n = 5.1(0.98)^{3n}$		
	$\frac{1.103^n}{(0.98^3)^n} = \frac{5.1}{2.5}$		
	$\left(\frac{1.103}{0.9412}\right)^n = 2.04$	M1	
	$(1.172)^n = 2.04$		
	$n \log 1.172 = \log 2.04$	M1	$n \log 1.172 = \log 2.04$
	$n = \frac{\log 2.04}{\log 1.172}$		
	$= 4.49$		
	$= 4.5$ years	A1	
		10	

No.	Mathematics ALT A paper 2	Marks	Comments
22.	(a)		

U.C.B	4.5	9.5	14.5	19.5	24.5	29.5
C.F	4	11	22	36	45	50

B1



✓

S1 ✓ Linear scale

P1 ✓ Plotting

C1 ✓ Curve

(b) (i) Median distance = distance covered by 25th teacher
= 15.5

B1

(ii) No of teachers in range $15 \leq d \leq 23$

No. of teachers $(d \leq 23) = 43$
No. of teachers $(d \geq 15) = 23$

B1 Any ✓

No. of teachers = (43 − 23) + 1

M1

= 21

A1

(c) No. of teachers $(d \leq 10) = 12$

B1

No of teachers $(d \leq 10)$ who used bike

$$= \frac{75}{100} \times 12$$

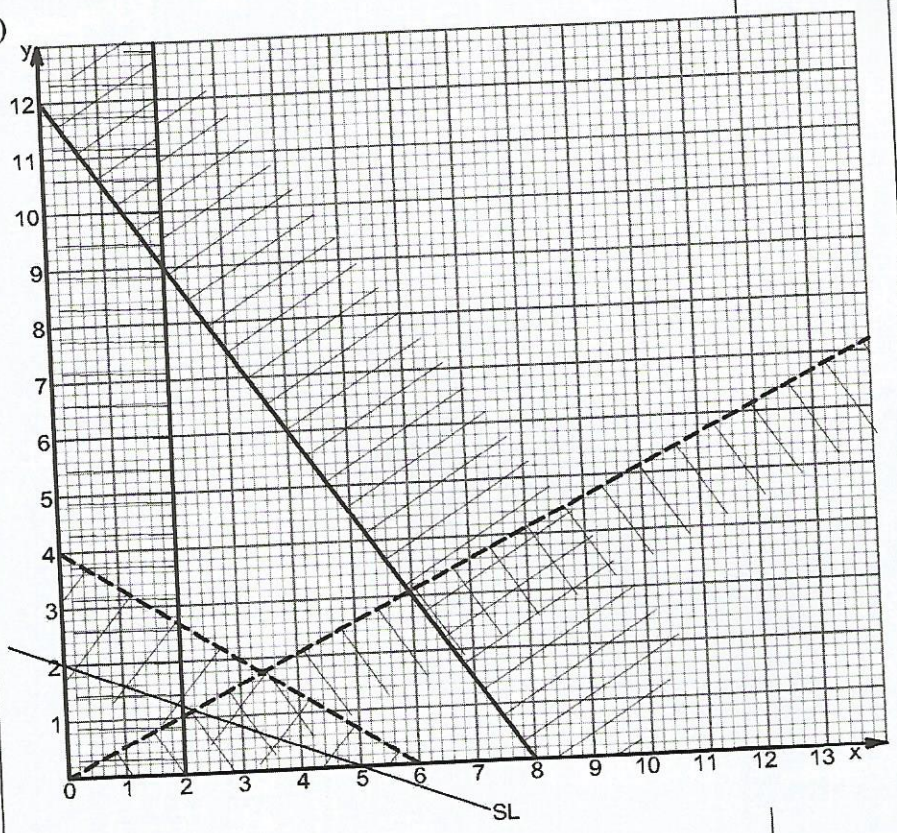
$$= 9$$

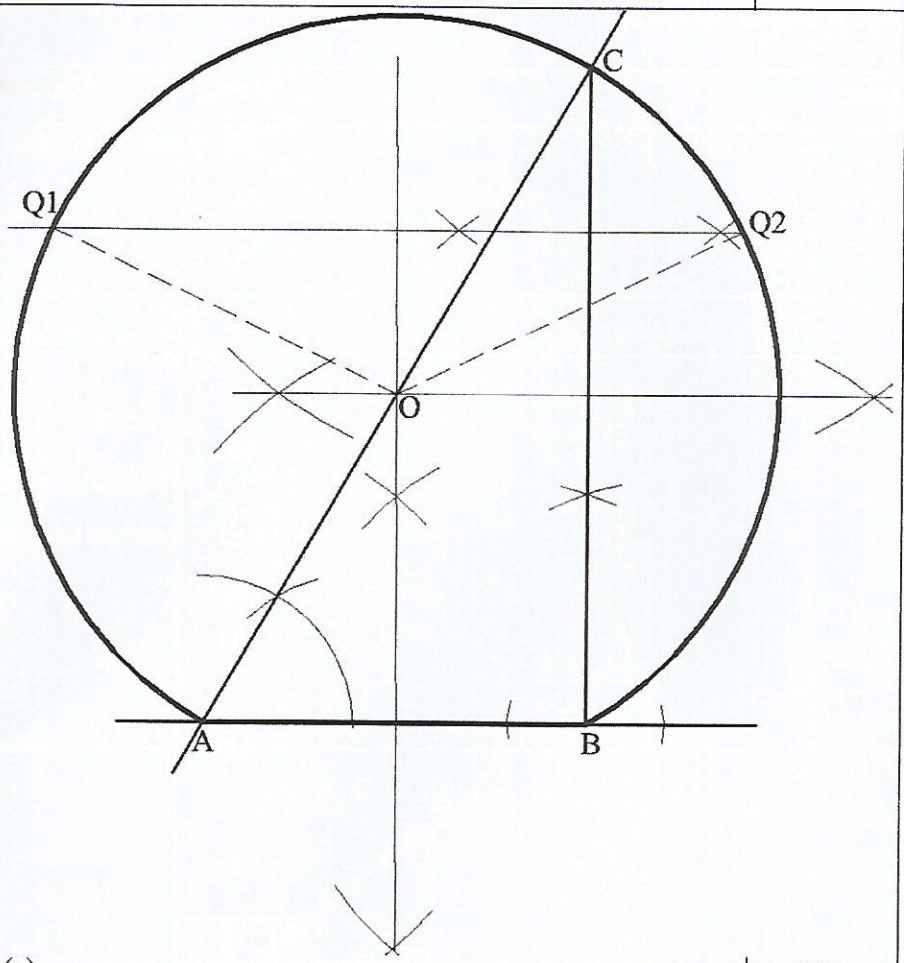
Amount raised $= 9 \times 50$

= Ksh 450

B1

10

No.	Mathematics ALT A paper 2	Marks	Comments
23.	(a) Inequalities satisfying information		
	$x \geq 2$	B1	
	$y > \frac{1}{2}x$	B1	
	$6x + 4y \leq 48$ or $3x + 2y \leq 24$	B1	
	$2000x + 3000y > 12000$ or $2x + 3y > 12$	B1	
	b) 		
		B1	$x \geq 2$
		B1	$y > \frac{1}{2}x$
		B1	$3x + 2y \leq 24$
		B1	$2x + 3y > 12$
		B1	
	(c) Equation of search line is $2x + 5y = 10$	B1	
	For maximum profit	B1	
	$x = 2,\ y = 9$		
		10	

No.	Mathematics ALT A paper 2	Marks	Comments
24.			
	(a)	B1 B1	√ Construction of 90° at B √ Construction of 60° at A and completion of Δ ABC
	(b)	B1 B1	√ location of centre O √ circumcircle
	(c)	B1 B1	Line // to AB at $\frac{3}{4}$BC Points Q_1 and Q_2 located
	$Q_1Q_2 = 9 \pm 0.1$ cm	B1	
	(d) Area above line Q bounded by locus P $\angle Q_1OQ_2 = 129° \pm 1°$ $r = 5$ cm	B1	
	$\text{Area} = \frac{129}{360} \times \pi \times 25 - \frac{1}{2} \times 25 \sin 129$	M1	
	$= 18.43\ \text{cm}^2$	A1 **10**	